

TRACK WIDTHS OF HEAVY IONS AND UNIT
MAGNETIC POLES

by

JESSE JAMES BUTTS, JR.

B. S., Kansas State University, 1961

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

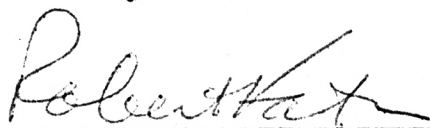
MASTER OF SCIENCE

Department of Physics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

Approved by:



Major Professor

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INTRODUCTION

An experiment designed to detect unit magnetic poles has just been concluded by Purcell (19). His is the most recent of a series of experiments stimulated by Dirac's work in 1931 with the quantum theory, predicting monopoles. Purcell looked for poles that might have been produced in proton nucleon interactions when 30 Bev protons were incident upon a target. The poles produced in this manner could have been collected and accelerated into a nuclear emulsion or some other detector. Using 30 Bev protons, the upper limit on the mass of the poles that might have been created was 3.1 amu. Undoubtedly, as larger accelerators become available, more experiments of this type will be done and the 3.1 amu limit on the pole mass can be pushed upward. Meanwhile, the logical place to look for monopoles is the cosmic rays. At altitudes of 100,000 feet or greater there are charged particles of every description possessing energies many orders of magnitude greater than are produced by the largest man-made machines. If poles are to be found anywhere, they will surely be in the cosmic rays.

Unlike neutrinos (particles long undetected because of their weak interaction with matter) monopoles interact with matter very strongly and therefore should be easy to detect. This property, combined with the fact that none have been observed, suggest that either poles are very rare, or they go about in disguise. According to Bauer (3) who has calculated the ionization of a monopole, "The track of a monopole, assumed heavy, in a detecting

device such as a photographic plate or cloud chamber would be similar to that of a very heavy nuclear fragment." Considering this possibility of confusion with heavy charges, which are known to be present in the cosmic rays, the purpose of this paper will be to discuss the tracks of charges and poles in electron sensitive photographic emulsion, and to outline a means of discrimination between them.

Aside from pole-charge discrimination, there is the problem of discrimination between heavy particles with slightly different charge. At present there is no easy or accurate way to identify the heavy charges found in the cosmic rays. If we can find a theory that will explain accurately some parameter of the tracks of charged particles, accurately enough to determine the charge Z , then upon translation to poles, this theory will enable us to discriminate between charges and poles. We will then first consider the problem of charge identification, and later translate our results to poles.

IDENTIFICATION OF HEAVY ION TRACKS

For particles with small charge, for example, protons, alpha particles, and lithium nuclei, there are standard methods of identification. These are grain counting, blob counting, gap measurement, combined with knowledge of range and/or multiple coulomb scattering. Since heavy particles leave solid tracks and scatter very little, other methods must be used.

At present the most widely used procedure for determining Z seems to be visually counting delta rays. This involves

counting the delta rays along the particle path, counting according to a minimum range, or minimum grain criterion. Delta ray density is a sensitive function of Z , but in practice is difficult to determine accurately and objectively. Some of the problems involved are discussed by Voyvodic (22). Among these are difficulty in the determination of counting efficiency, minimum energy delta ray counted, and variation of these two factors with range. Furthermore these will vary with the individual doing the counting resulting in a subjective parameter at best.

There is another delta ray technique presently being used by Alvial (1), who measures the length, parallel to the particle axis, of short dense projections from the main body of ionization. This length is divided by the mean grain diameter to determine the number of "little delta rays" present.

The thin down length or range of maximum track width has been used (13) to identify heavy charges. This parameter is an insensitive function of Z , and is also difficult to measure accurately, see Plates VIII through XI.

A parameter used by some investigators (4), (10), (16), (21), and which will be used exclusively in this paper is the track width. The track width as a function of range goes through a maximum for a charged particle. In the thin down region near the end of a heavy track, the track width decreases even though the ionization is still increasing. The observed decrease in width was originally attributed to electron pickup as the ion slowed near the end of its path (11), but this theory was later disproved

by Lonchamp's experiments (16). Lonchamp, by using heavy ions from an accelerator, found that the maximum width occurred at a much larger range than could be explained by the electron pickup theory. This led him to propose a theory of width based upon the spatial distribution of delta rays along the track. According to Lonchamp, electron pickup is a relatively small effect, and the track thins down mainly because the maximum range of the delta rays in the low energy region is approaching zero. Lonchamp's theory has been extended to poles by Katz and Parnell (14) to predict width as a function of the particle velocity, βc . This extension is repeated here and carried a step farther to obtain width as a function of range, an observable quantity.

Bizzeti and Della Corte (4) have developed a theory of track width that they used in the thin down region. Their theory is basically the same as Lonchamp's, but is considerably more refined and more complicated. However, their data is taken only in the last 300 microns and therefore their theory covers only the thin down region of track. Since we are interested in the track width at all ranges in connection with monopole identification, we have extended their theory to the high energy region. The extended theory was found to be in approximate agreement with experimentally observed widths up to ranges of 1,000 microns. This extended theory, which seems to work well for charges, has been extended to monopoles.

THEORIES OF TRACK FORMATION

Lonchamp's Theory

Lonchamp (16) in 1953 proposed a theory for track width which gives qualitative if not exactly quantitative agreement with experimental data.

Lonchamp assumed that the track width will be $2r$ where r is the radius of a cylinder whose axis is the particle's path and whose surface is intersected by n normally ejected rectilinear delta rays per unit length of primary path. That is, if there are n delta rays per 100 microns with range greater than r , the track width will be $2r$.

The number of delta rays with energy between W and $W+dW$, ejected by a heavy particle with charge Ze and velocity βc , is given by Rutherford's formula,

$$dn = \frac{2\pi N e^4}{m_0 c^2} \frac{Z^2}{\beta^2} \frac{dW}{W^2}, \quad (1)$$

where N is the number of electrons per unit volume, e is the electronic charge, and m , the electron mass. If we integrate this formula between the limits W_0 and W_{\max} we have

$$n = \frac{2\pi N e^4}{m_0 c^2} \frac{Z^2}{\beta^2} \left[\frac{1}{W_0} - \frac{1}{W_{\max}} \right] \quad (2)$$

Now if we evaluate the constant and convert from "per unit length"

to "per 100 microns", we obtain

$$M = 5.34 \times 10^{-3} \frac{Z^2}{\beta^2} \left[\frac{510}{W_0} - \frac{1}{2\beta^2} \right], \quad (3)$$

where W_{\max} has been set equal to $2m_0c^2\beta^2$ the maximum energy transferred to an electron by a heavy particle with velocity βc . Equation (3) gives the number of delta rays, n , with energy greater than W_0 per 100 microns of track. If we assume that when n_0 delta rays pass out of the cylinder there will be development of enough grains inside to render the cylinder completely opaque in projection, then we can replace n by n_0 in Equation (3) and solve for W_0 as a function of β giving

$$W_0 = \frac{1020}{\frac{m_0 375 \beta^2}{Z^2} + \frac{1}{\beta^2}} \quad (4)$$

as the energy of a delta ray whose range is the cylinder radius or one-half the track width. Now if we assume that the delta rays obey the range equation given by Katz and Penfold (15)

$$r = R W^{1.72} = 2.1 \times 10^{-2} W^{1.72} \quad (5)$$

where r is in microns and w in keV, we can calculate the track width, $2r$, by replacing w by the value of W_0 given in Equation (4) to obtain

$$\text{Width} = 2r = 2(2.1) \times 10^{-2} \left[\frac{1020}{\frac{m_0 375 \beta^2}{Z^2} + \frac{1}{\beta^2}} \right]^{1.72} \quad (6)$$

But
$$\beta^2 = 1 - \left(\frac{M_0 c^2}{E + M_0 c^2} \right)^2$$

where E is the kinetic energy and $M_0 c^2$ is the rest mass energy of the particle. We then have

$$\text{Width} = 4.2 \times 10^{-2} \left[\frac{\frac{1020}{375 M_0 \left[1 - \left(\frac{M_0 c^2}{E + M_0 c^2} \right)^2 \right]}}{Z^2} + \frac{1}{1 - \left(\frac{M_0 c^2}{E + M_0 c^2} \right)^2} \right]^{1.72} \quad (7)$$

Using Demers (8) range energy curves for charged particles, values of width have been calculated for various ranges and Z 's, Plate I. Width versus range for $Z=14$ using three different values of n_0 is plotted in Plate II.

Theory of Bizzeti and Della Corte

The following theory is given by Bizzeti and Della Corte (4) for the width in the thin down region.

It is postulated that if the energy flux per unit area out of a cylinder of radius x , whose axis is the particle's path, exceeds a given constant value E^* , then the track width will be given by

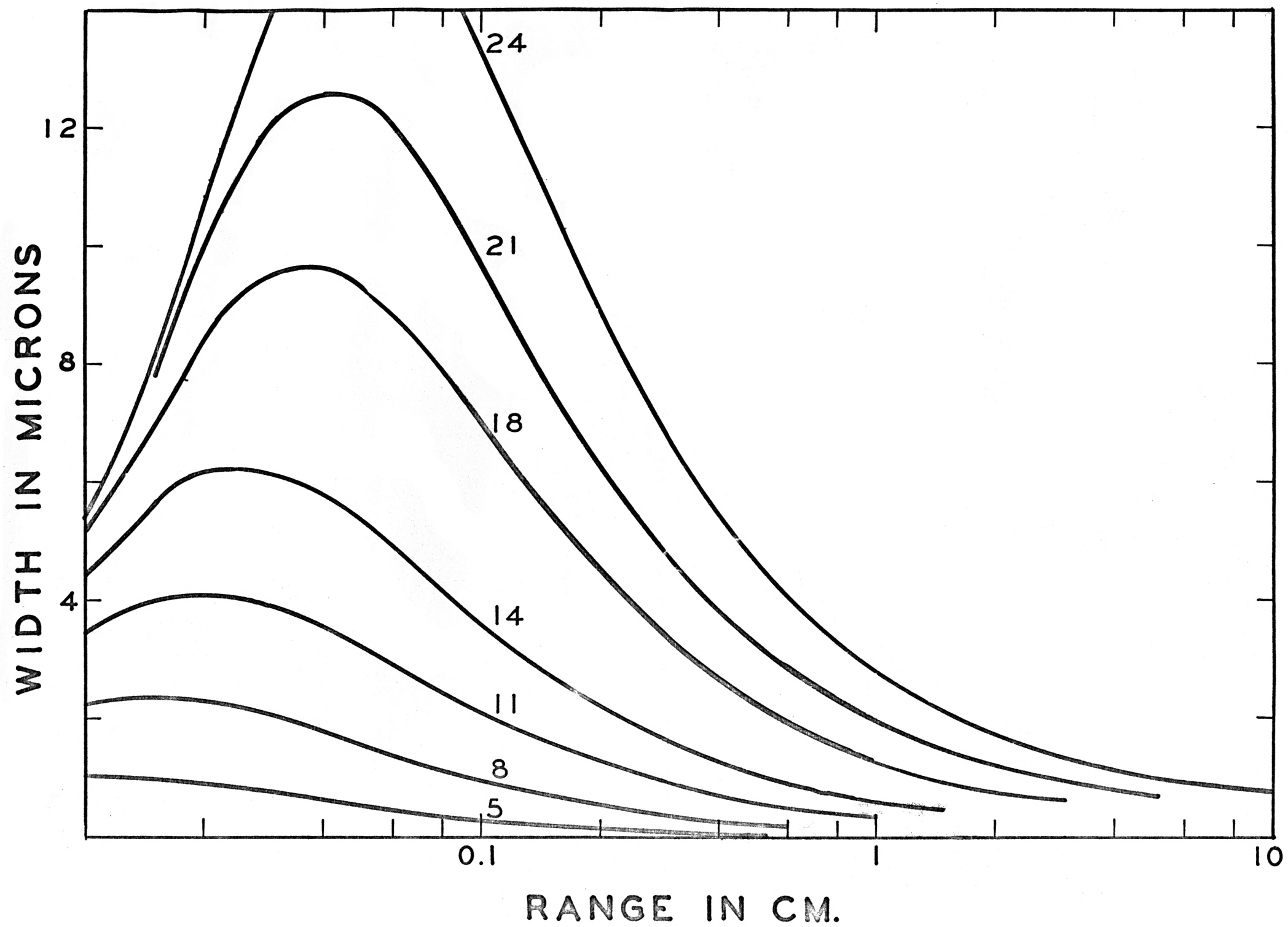
$$\lambda = 2x + \lambda_0 = \lambda_1 + \lambda_0 \quad (8)$$

where λ_0 is a constant that depends only upon the development of the emulsion. It can be seen by considering the geometry before and after development, Plate III, Figure 1, that $\lambda_0 = (g+1)d$ where

EXPLANATION OF PLATE I

Theoretical width versus range for seven
values of Z (Lonchamp's theory).

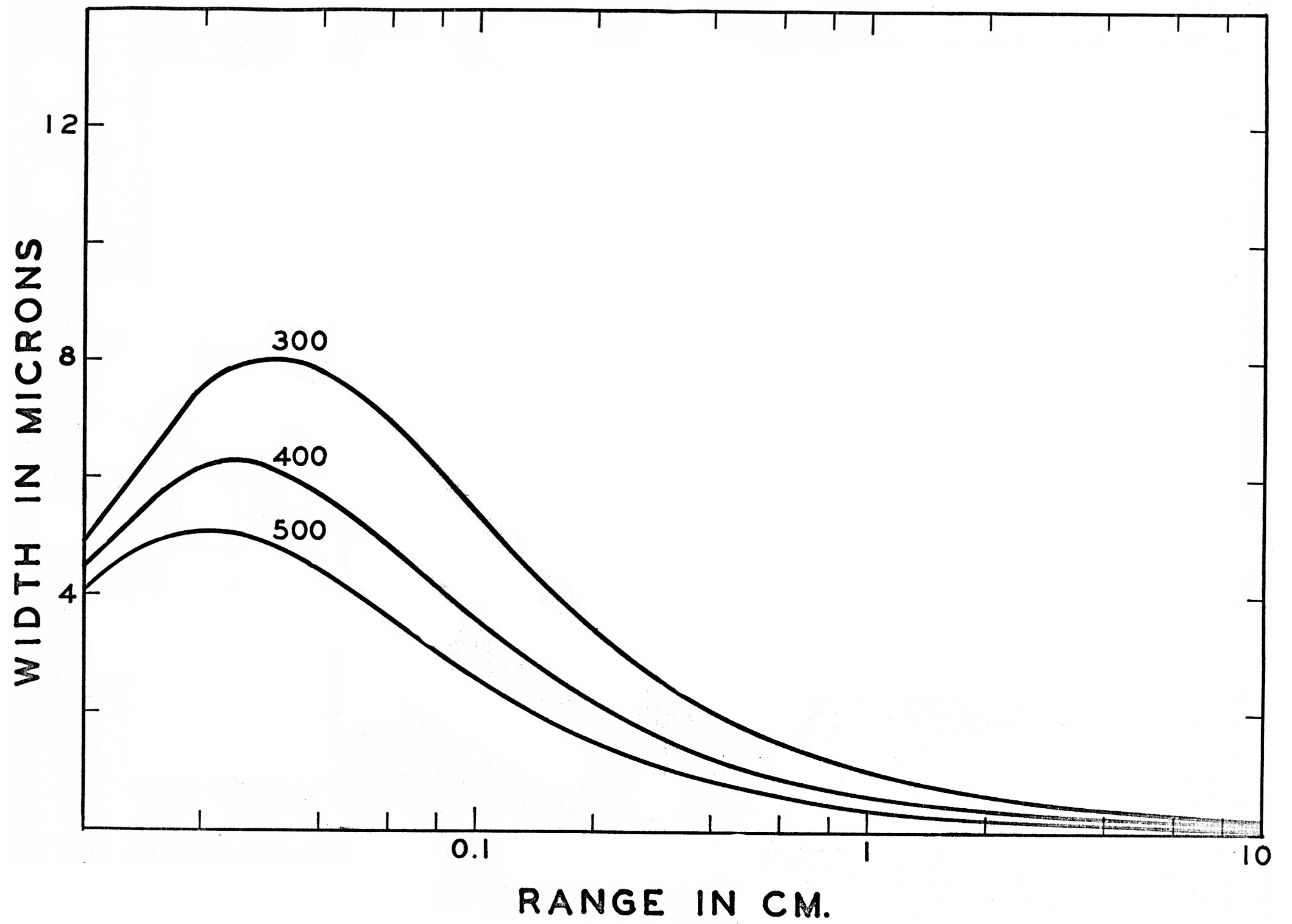
PLATE I



EXPLANATION OF PLATE II

Theoretical width versus range for $Z = 14$ and
 n_0 300, 400, 500. (Lonchamp's theory).

PLATE II



d is the diameter of an average grain before and gd is the diameter after development.

The hypothesis then amounts to requiring that $E(x)$, the energy transferred by secondary electrons, per unit length of primary path, outside a cylinder of radius x , satisfies the condition that

$$\frac{E(x)}{2\pi x} = E^* = \text{const.} \quad (9)$$

Now if $\bar{W}(x, w)$ is the average amount of energy deposited outside a cylinder of radius x by a delta ray of energy w , then the total energy deposited outside this cylinder will be

$$E(x) = \int_{W(x)}^{W_{\max}} \bar{W}(x, w) dn \quad (10)$$

where dn is the number of delta rays with energy between W and $W+dW$ and, as before, is given by

$$dn = \left(\frac{2\pi N e^4}{m_0 c^2} \right) \frac{z^2}{\beta^2} \frac{dW}{W^2} = K \frac{z^2}{\beta^2} \frac{dW}{W^2} \quad (1)$$

We have then

$$E(x) = K \frac{z^2}{\beta^2} \int_{W(x)}^{W_{\max}} \frac{\bar{W}(x, w)}{w} \frac{dw}{w} \quad (11)$$

where $W_{\max} = 2m_0 c^2 \beta^2 \gamma^2$ is the maximum energy of a delta ray produced by a particle with velocity βc ; $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$.

Now if the range energy relation for delta rays is assumed to be as before,

$$r = k w^{\alpha} = 2.1 \times 10^{-2} w^{1.72} \quad (5)$$

we can make a change of variable and obtain

$$E(x) = \frac{k}{\alpha} \frac{x^2}{\beta^2} \int_x^R \frac{\bar{W}(x,r)}{W(r)} \frac{dr}{r}, \quad W(r) = \left(\frac{r}{k}\right)^{\frac{1}{\alpha}} \quad (12)$$

where $R = k(W_{\max})^{\alpha}$ is the maximum range of a delta ray.

Now to proceed we must evaluate the integral in Equation (12). To do this we assume that the delta rays follow straight line paths and that their angular distribution is energy-independent.

The energy deposited outside of a cylinder of radius x by a single delta ray will be given by

$$w(r, x, \theta) = k^{-\frac{1}{\alpha}} \left(r - \frac{x}{\sin \theta}\right)^{\frac{1}{\alpha}} \quad (13)$$

with

$$\theta_0 \leq \theta \leq \pi - \theta_0, \quad \theta_0 = \sin^{-1}\left(\frac{x}{r}\right)$$

where $r = k w^{\alpha}$ is the range, and θ is the angle of ejection of the electron, see Plate III, Figure 2. Now, we average over the angle variable, θ , to obtain

$$\bar{W}(r, x) = k^{-\frac{1}{\alpha}} \int_{\theta_0}^{\pi - \theta_0} \left(r - \frac{x}{\sin \theta}\right)^{\frac{1}{\alpha}} f(\theta) 2\pi \sin \theta d\theta \quad (14)$$

EXPLANATION OF PLATE III

Figure 1. Crossectional view of track showing undeveloped grain of diameter D . Before development the grain is just touching the cylinder of radius x . After development the grain has grown to a diameter GD and extends inside the cylinder. Total track width after development is $\lambda = 2\psi + (G+1)D$.

Figure 2. Longitudinal view of track showing delta ray of range r and ejection angle θ , penetrating the cylinder of radius x .

PLATE III

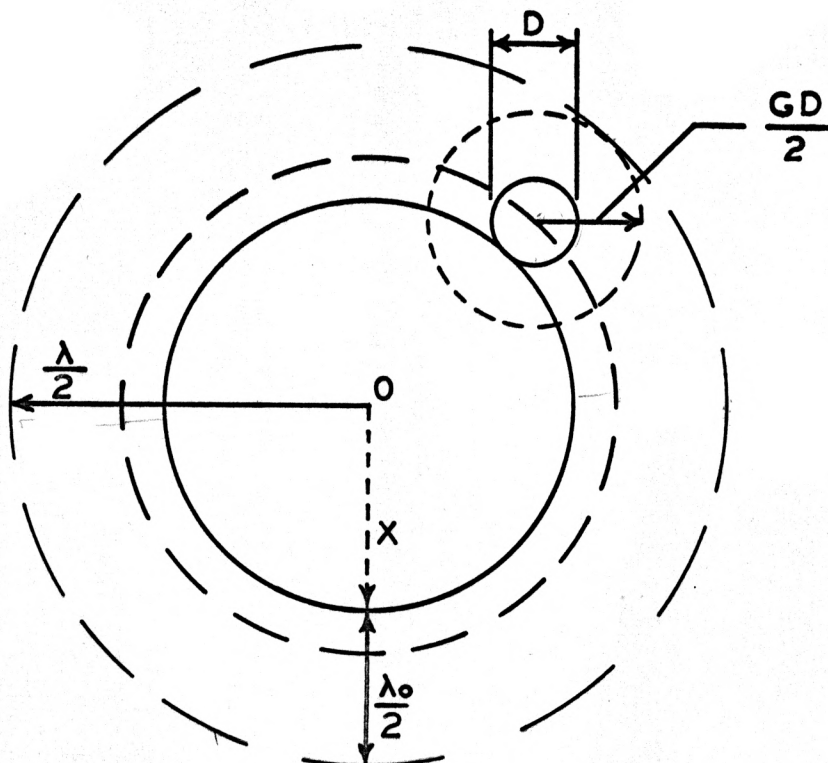


FIGURE 1. SECTION OF TRACK

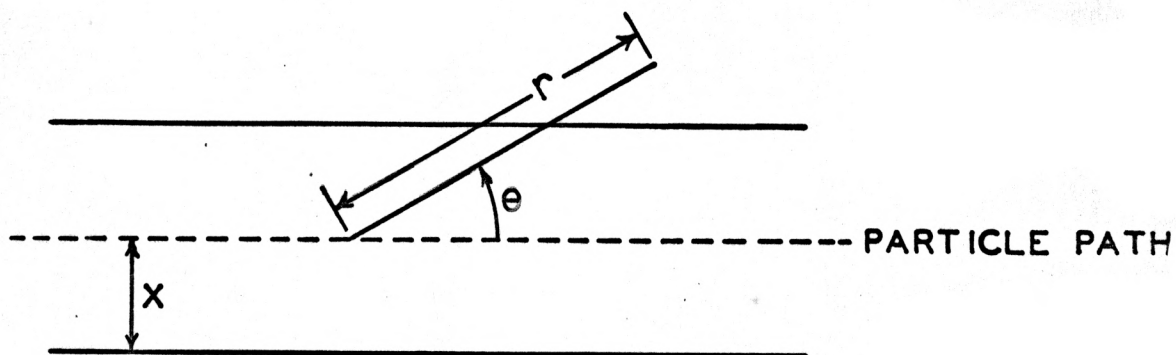


FIGURE 2. DELTA RAY EJECTION

where $2\pi \sin\theta d\theta$ is the solid angle available at the angle θ . The probability density of θ is $f(\theta)$ and is assumed to be independent of the delta ray energy. We now write the integrand in Equation (12) as

$$\frac{\bar{W}(r, z)}{W(r)} = \int_{\theta_0}^{\pi-\theta_0} \left(1 - \frac{r}{r \sin\theta}\right)^{\frac{1}{\alpha}} f(\theta) 2\pi \sin\theta d\theta \quad (15)$$

or,

$$\frac{\bar{W}(r, z)}{W(r)} = \int_{\theta_0}^{\frac{\pi}{2}} \left(1 - \frac{r}{r \sin\theta}\right)^{\frac{1}{\alpha}} \{f(\theta) + f(\pi-\theta)\} 2\pi \sin\theta d\theta. \quad (16)$$

If we make the assumption that

$$f(\theta) + f(\pi-\theta) = \frac{1}{2\pi} \quad (17)$$

we have

$$\frac{\bar{W}(r, z)}{W(r)} = \int_{\theta_0}^{\frac{\pi}{2}} \left(1 - \frac{r}{r \sin\theta}\right)^{\frac{1}{\alpha}} \sin\theta d\theta = \frac{1}{z} \int_1^z \left(1 - \frac{1}{y}\right)^{\frac{1}{\alpha}} \frac{y dy}{\sqrt{z^2 - y^2}} \quad (18)$$

where $z = \frac{r}{R}$ and $y = \frac{r \sin\theta}{R}$. Now we are able to calculate the integral that appears in Equation (12)

$$I\left(\frac{r}{R}\right) = \int_r^R \frac{\bar{W}(r, r)}{W(r)} \frac{dr}{r} = \int_1^{\frac{R}{r}} \frac{dz}{z} \frac{1}{z} \int_1^z \left(1 - \frac{1}{y}\right)^{\frac{1}{\alpha}} \frac{y dy}{\sqrt{z^2 - y^2}} = \int_1^{\frac{R}{r}} \left(1 - \frac{1}{y}\right)^{\frac{1}{\alpha}} y dy \int_y^{\frac{R}{r}} \frac{1}{\sqrt{z^2 - y^2}} \frac{dz}{z^2},$$

and

$$I\left(\frac{r}{R}\right) = \int_1^{\frac{R}{r}} \left(1 - \frac{1}{y}\right)^{\frac{1}{\alpha}} \sqrt{1 - \left(\frac{ry}{R}\right)^2} \frac{dy}{y}. \quad (19)$$

This integral has been evaluated numerically.

We are now able to combine the three equations

$$\lambda = 2\mu + \lambda_0 \quad \text{or} \quad \mu = \frac{\lambda - \lambda_0}{2}, \quad (8)$$

$$\frac{E(\mu)}{2\pi\mu} = E^* \quad (9)$$

$$E(\mu) = \frac{K}{\alpha} \frac{Z^2}{\beta^2} I\left(\frac{\mu}{R}\right) = \frac{K}{\alpha} \frac{Z^2}{\beta^2} \int_{\mu}^R \frac{W(\mu, r)}{W(r)} \frac{dr}{r} \quad (12)$$

to obtain

$$\frac{Z}{\beta\sqrt{R}} = \phi\left(\frac{\lambda - \lambda_0}{2R}\right) = \left[\frac{2\pi\alpha E^*}{K} \frac{\mu}{R} \frac{1}{I\left(\frac{\mu}{R}\right)} \right]^{\frac{1}{2}} \quad (20)$$

This relation enables us to plot $\lambda - \lambda_0/2R$ vs. $\frac{Z}{\beta\sqrt{R}}$, Plate IV.

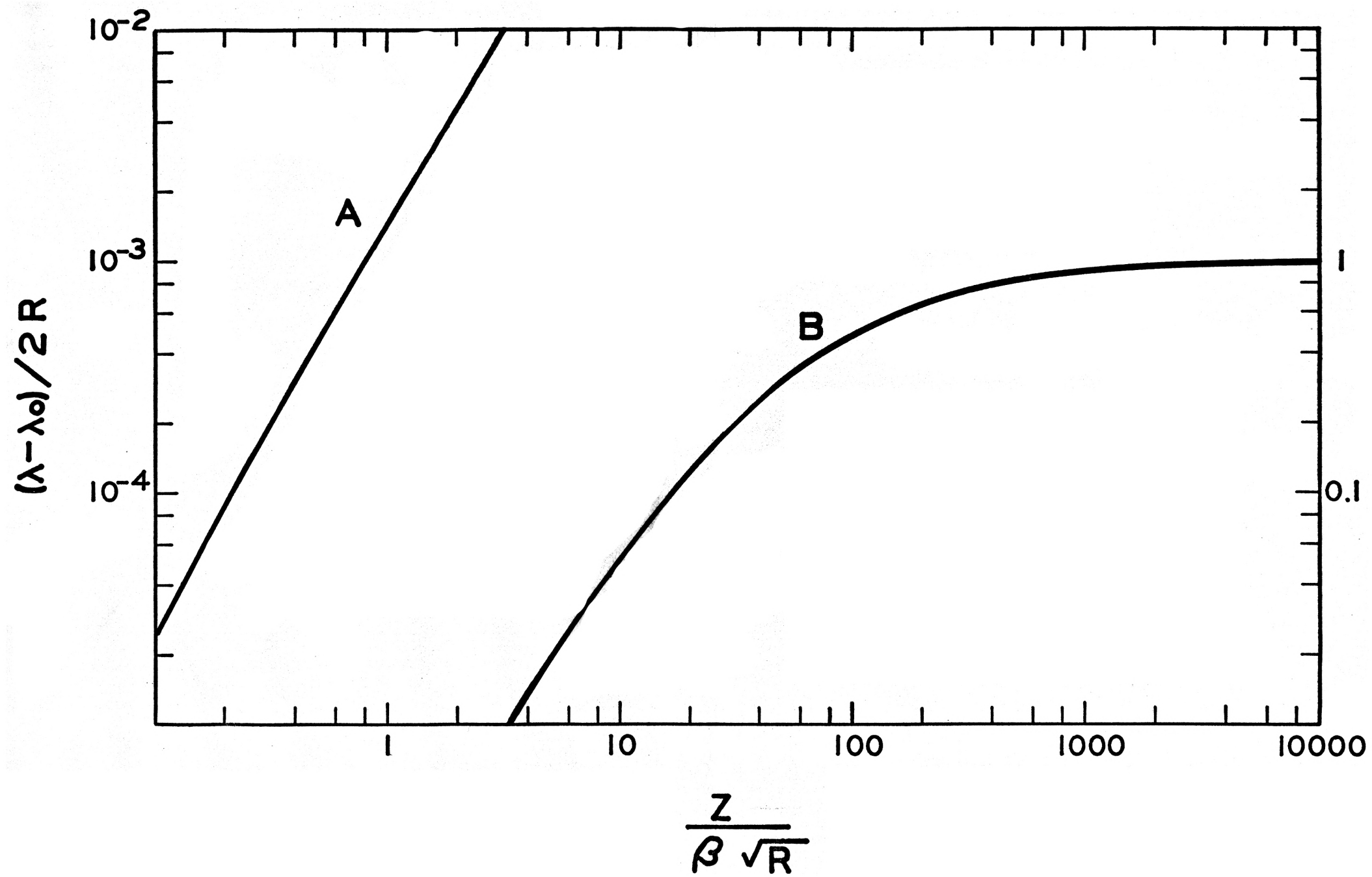
The next step is to use range energy curves (8) or an appropriate range energy formula for heavy ions and calculate β for the heavy ions and R , the maximum range of the delta ray at this β , for various ranges and by using the curve in Plate IV we can plot $\lambda - \lambda_0$ versus range for various Z 's, Plate V.

Note that the only two adjustable parameters are λ_0 and E^* . E^* was taken to be 9.8 keV per square micron, a value chosen to

EXPLANATION OF PLATE IV

Plot of $\lambda - \lambda_0 / 2R$ versus $z / \beta \sqrt{R}$. Vertical scale for Curve A is on the left. Curve B is the continuation of Curve A and is referred to the vertical scale on the right.

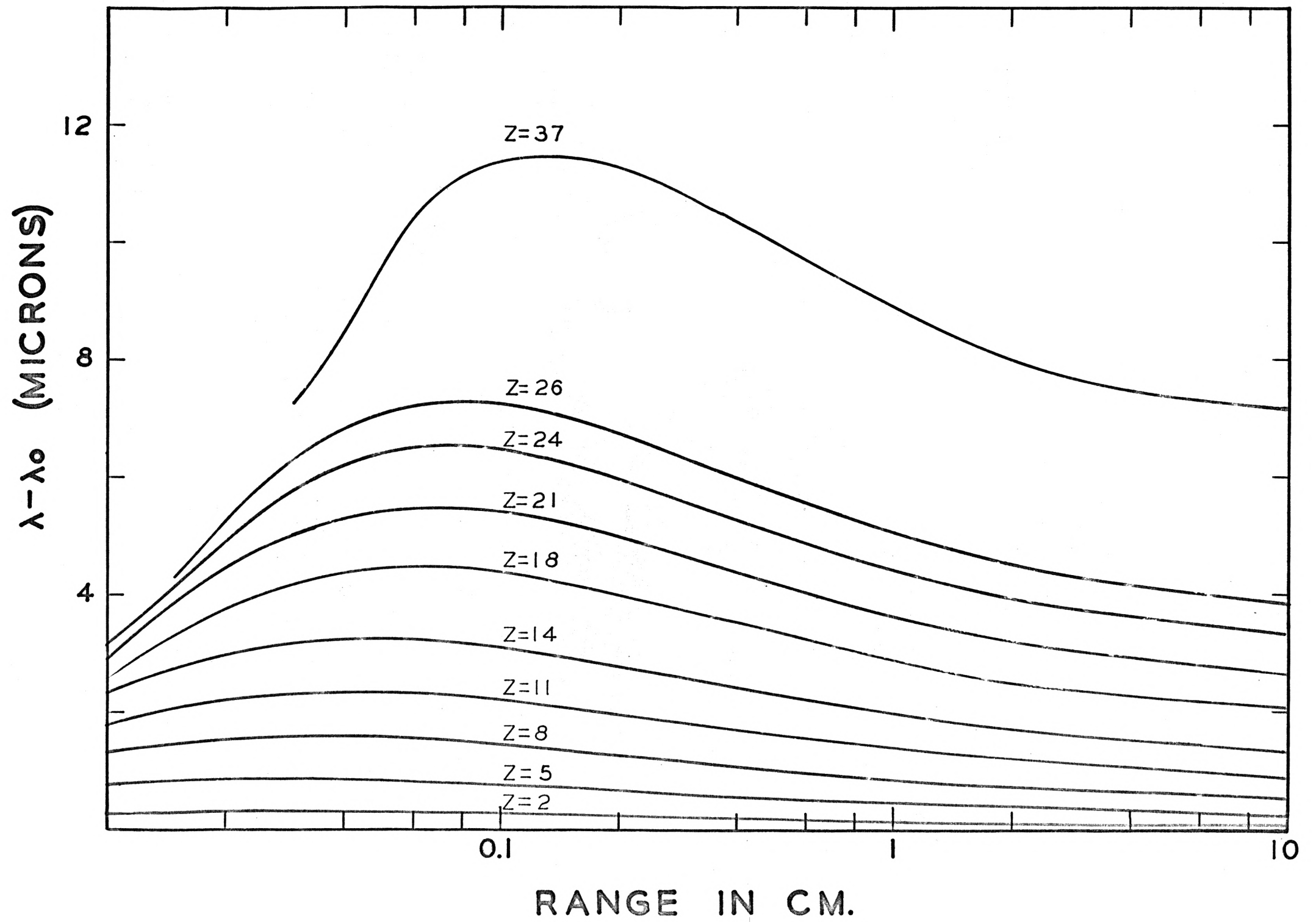
PLATE IV



EXPLANATION OF PLATE V

Semilog plot of $\lambda - \lambda_0$ (microns) versus Residual Range (cm)
for ten values of Z. (Bizzeti and Della Corte's Theory).

PLATE V



fit the theory to the experimental data Bizzeti and Della Corte give for accelerated charges (4).

EXPERIMENTAL PROCEDURES AND RESULTS

The actual width of a track is statistical in that it varies randomly about a mean value for short increments of range. To smooth out this roughness we will define the width to be the projected area of a small length divided by that length. The length chosen must be small enough to insure that the mean width is constant in the interval but large enough to obtain the desired smoothing. This definition of track width is by no means the only one used. For a comprehensive study of the various methods of track width measurement, see Gegauff (12).

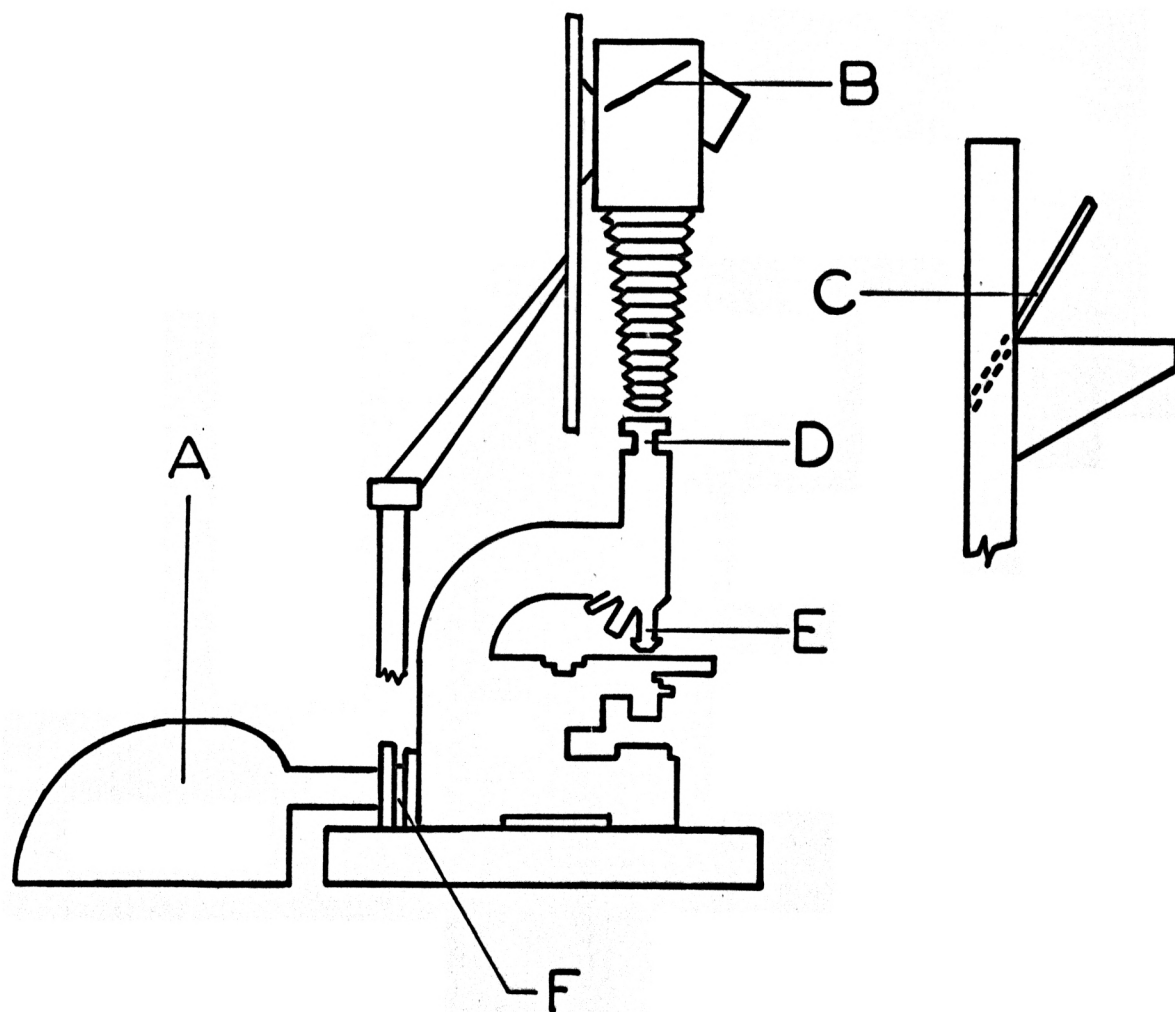
To measure the area, a screen consisting of a drawing board covered with graph paper, was placed in front of a Leitz Ortholux microscope with Aristophot bellows extension and reflex camera attachment and the image of the track focused on it, see Plate VI. Careful attention was given to the arrangement of the screen in order to obtain a distortion free image. This arrangement gave a 3360x magnification when a 100x Ks oil immersion objective and a 6x eyepiece were used. The image projected in this manner was outlined in pencil and these outlines traced with a planimeter to obtain the area. The planimeter was found to be inaccurate for small areas so that a compromise was made between large areas and distinctness of image, resulting in the rather large (3360x) value of the magnification. In order to have sufficient light for tracing the image, and at the same time prevent heat damage to

EXPLANATION OF PLATE VI

Experimental apparatus consisting of

- A. Xenon lamp
- B. Reflex mirror
- C. Board covered with graph paper
- D. 6x eyepiece
- E. 100 x ks oil immersion objective
- F. Heat absorbing glass

PLATE VI



the emulsions, a xenon lamp in conjunction with many layers of heat absorbing glass was used, Plate VI.

The plate containing the track to be measured was placed on the stage in such a manner as to give a vertical image on the screen. Range measurements were then read directly off the stage micrometer dial. With this arrangement the length of track traced was 53 microns, all that could be seen in projection.

All tracks measured were very flat, and a calculation indicated that in neglecting to correct for dip angle, negligible error was made.

Width measurements were made on eight tracks found in a stack of emulsions exposed to the cosmic rays for a period of ten hours at approximately 100,000 feet altitude over Central Canada. The tracks measured were all heavy enough to be seen with the unaided eye and were selected for their length and small dip angle. They were all greater than one centimeter long and the maximum dip angle was 9.5 degrees. Measurements were made at 32 different ranges on each track and the results plotted on semilog graph paper.

Discontinuities were observed in the curves corresponding to places where the tracks leave one pellicle and enter another. These discontinuities which arise because of variation of grain size with depth indicated that a normalization to depth was required. To determine the normalization needed, a track of a fast heavy particle whose width would otherwise have been constant in these plates, was measured through two pellicles. The plot of

width versus depth for this track indicated that a simple normalization procedure was justified. The width measurements versus depth as well as the normalization curve used is given in Plate VII.

To apply Bizzeti's theory, the value of λ_0 must be determined. This was done by measuring the grain diameter, gd , at 10 micron depth intervals through the emulsion. Then, taking d , the undeveloped grain diameter, to be .27, a value given by Voyvodic (22), for Ilford G-5 emulsion, we calculated $\lambda_0 = (g+1)d$ to be .912 microns.

Using the normalization described above and $\lambda_0 = .912$, in Bizzeti's theory the width versus range for the eight tracks has been plotted over the theoretical curves in Plates VIII through XI.

DISCUSSION OF THEORY AND EXPERIMENTAL RESULTS

In trying to fit the data, Plates VIII through XI, to the theories, Plates I and V, we find that Bizzeti's theory explains the track width better than the simple theory of Lonchamp.

If we attempt to modify Lonchamp's theory in the high energy region where the theoretical width is too small, we must decrease n_0 to increase the width. This modification is hard to justify since in the high energy region there are more high energy delta rays which deposit most of their energy some distance from the track, and hence contribute little to the track width. Furthermore, if we attempt to make the maximum width correspond to an experimental maximum width, by adjusting n_0 , we find that the

EXPLANATION OF PLATE VII

Experimental points are width measurements made on track K26-1, going through Plates K22 and K23. The range varies from 21,500 microns to 33,700 microns, and is large enough that the track width over the interval should be a function of depth in the emulsion only.

The solid curve is used to normalize the experimental data to depth beneath the surface of the emulsion.

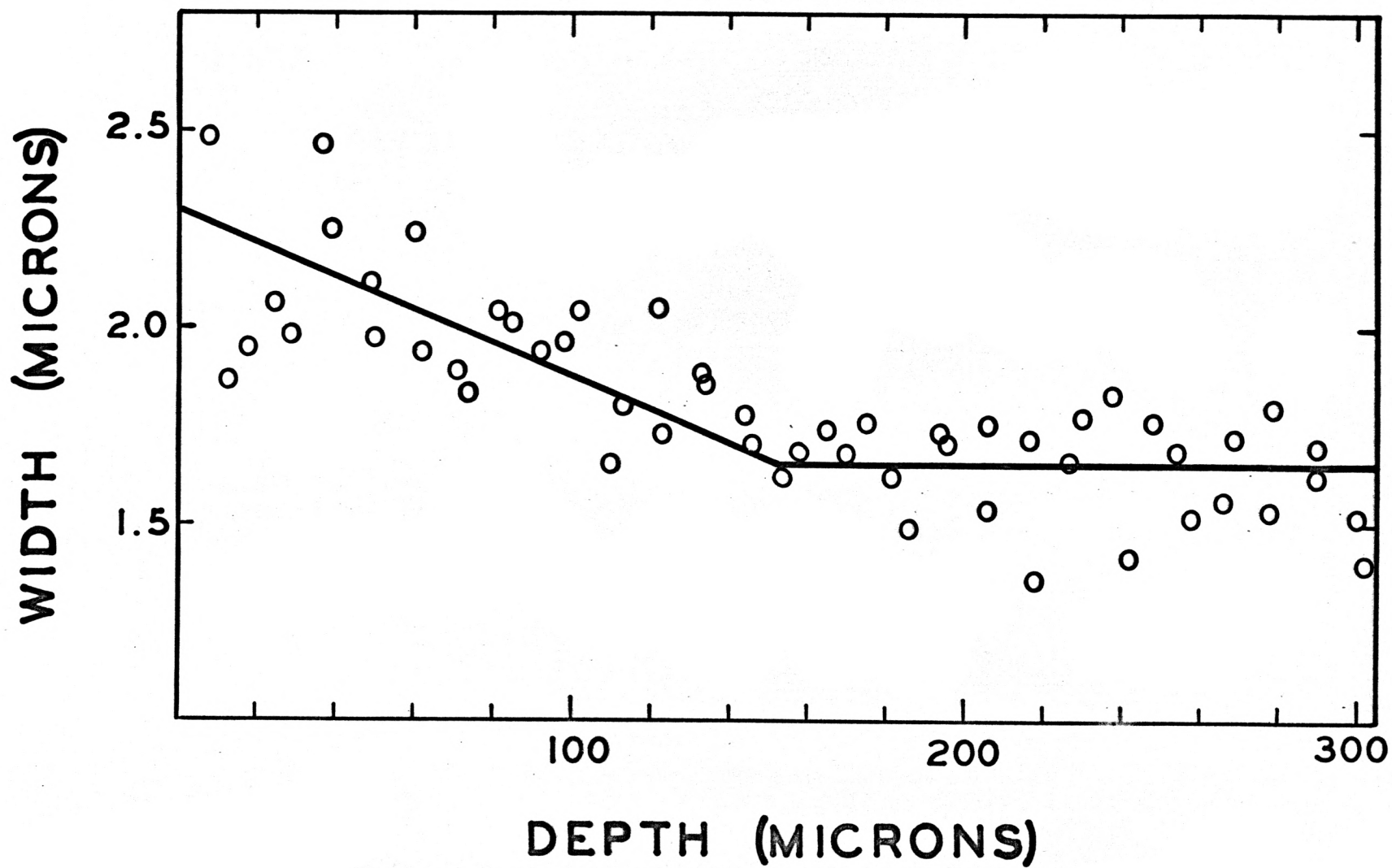
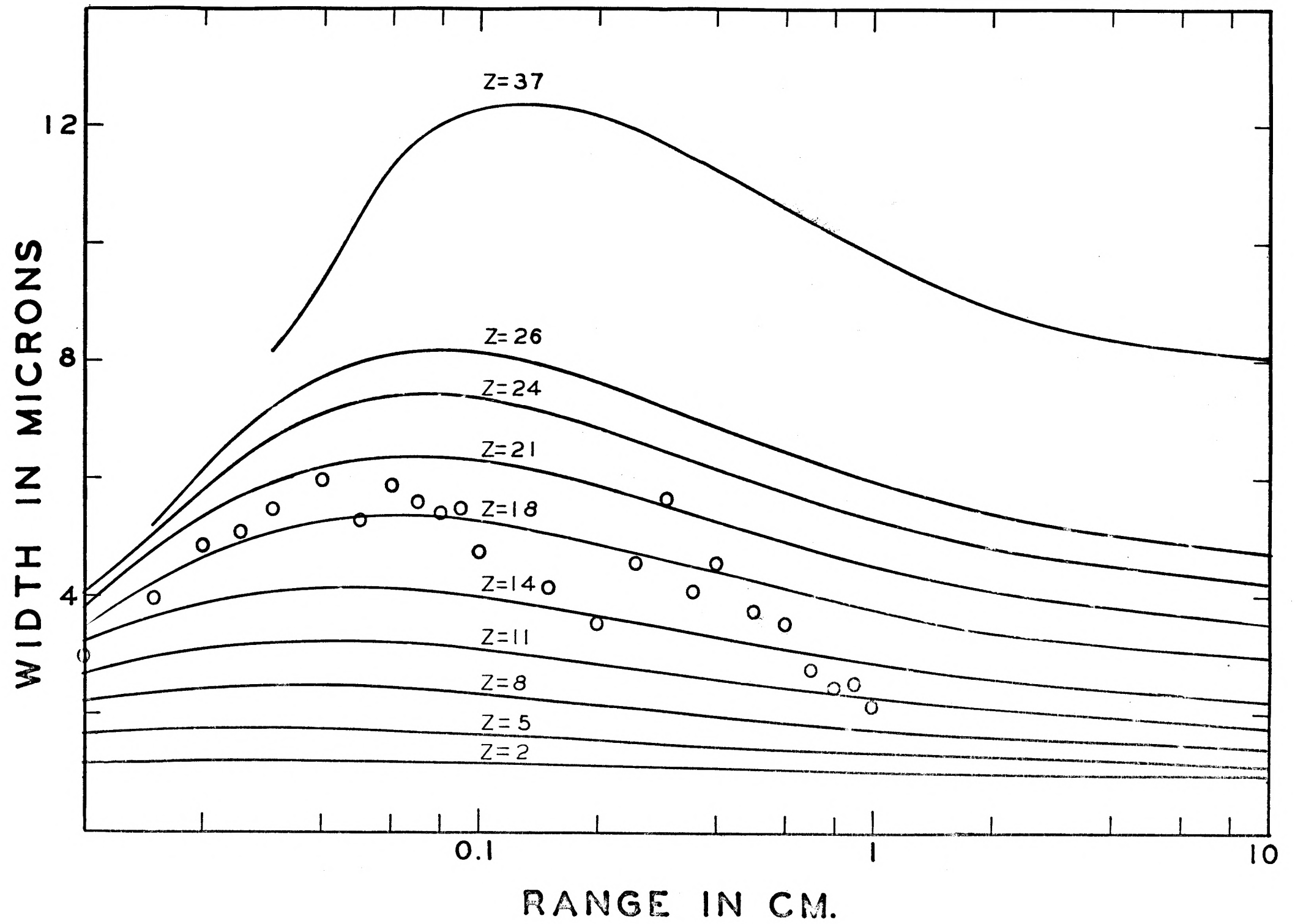


PLATE VII

EXPLANATION OF PLATE VIII

Experimental width versus range for track
K35-1 plotted over theoretical curves
of Bizzeti and Della Corte.

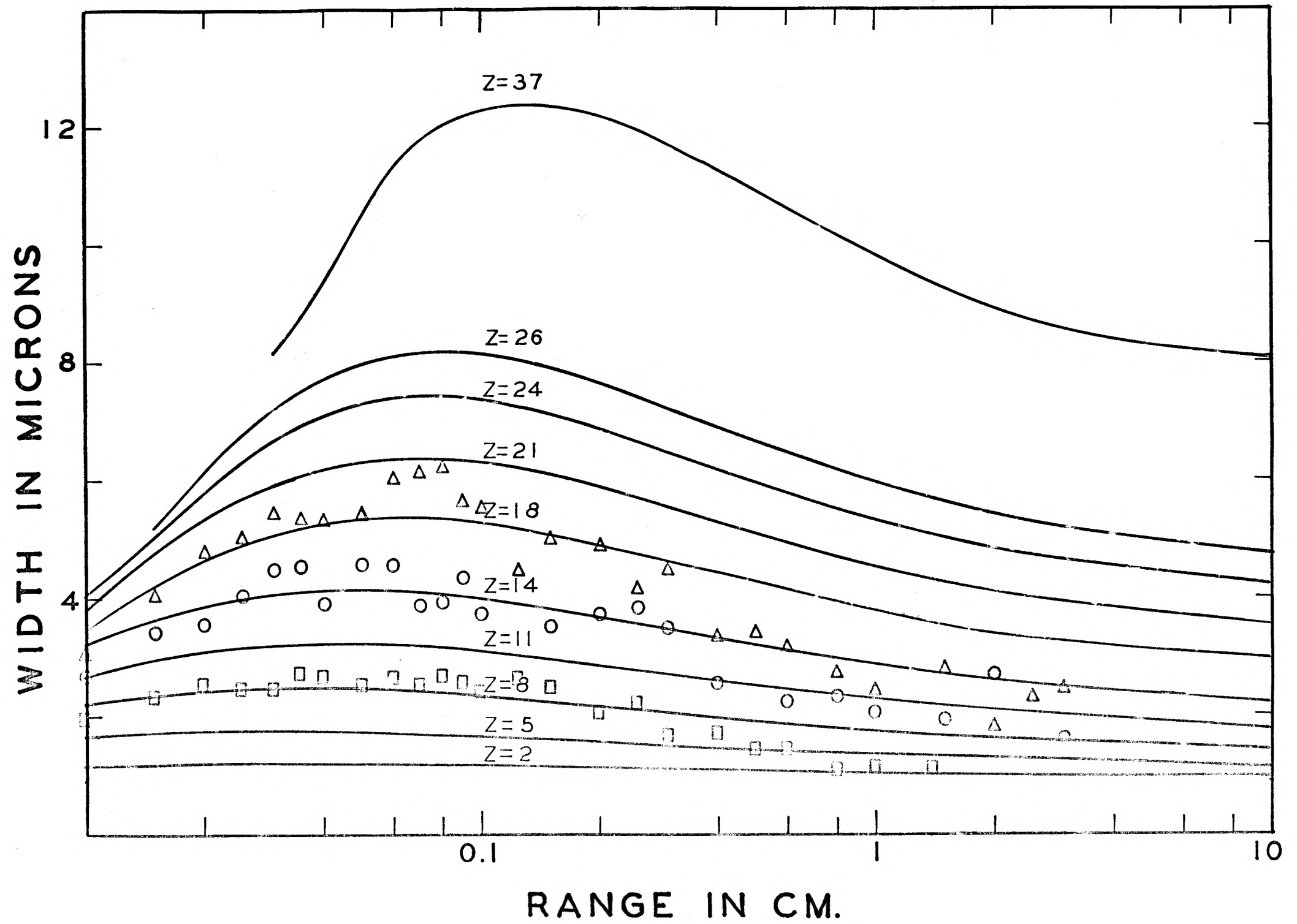
PLATE VIII



EXPLANATION OF PLATE IX

Experimental width versus range for tracks
K26-1 (o), K40-3 (\square), and K40-1 (\triangle) plotted
over theoretical curves of Bizzeti and
Della Corte.

PLATE IX

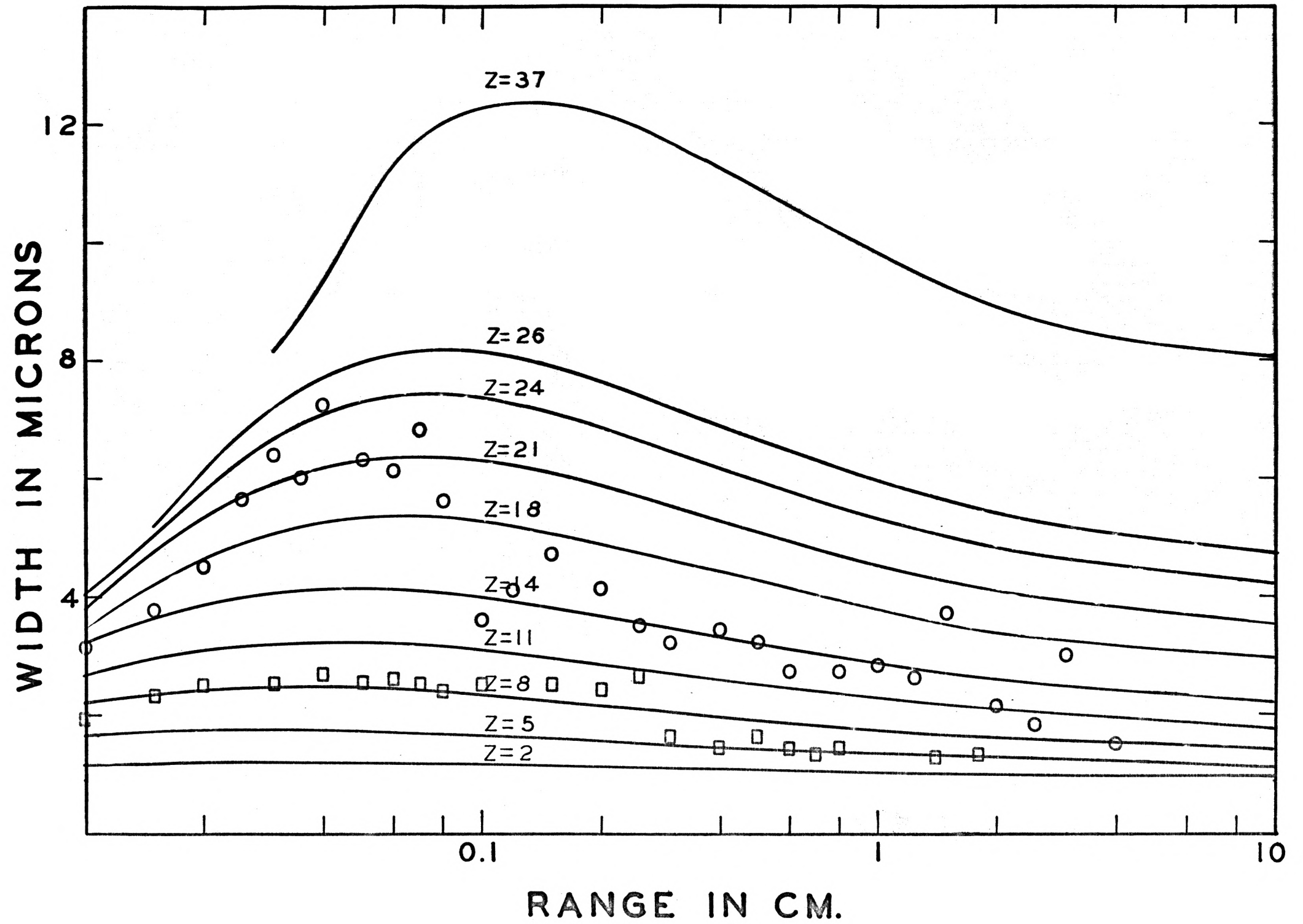


EXPLANATION OF PLATE X

Experimental width versus range for tracks

K34-1 (\square), and K40-2 (O) plotted over theoretical
curves of Bizzeti and Della Corte.

PLATE X

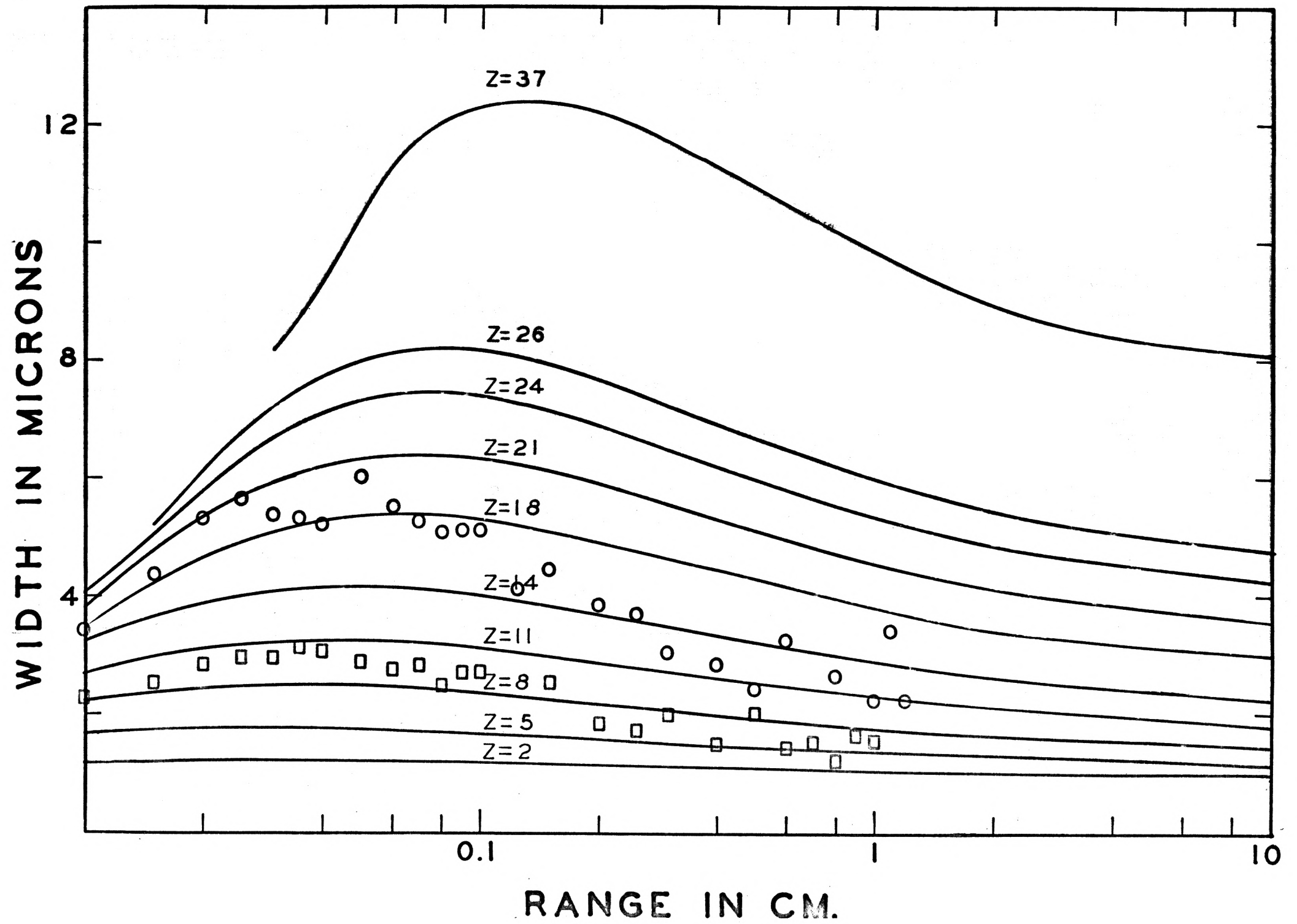


EXPLANATION OF PLATE XI

Experimental width versus range for tracks

K36-2 (O), and K36-1 (\square) plotted over theoretical
curves of Bizzeti and Della Corte.

PLATE XI



range at which this maximum occurs is much too small, see Plate II.

The principle assumption of Bizzeti is that the energy flowing out of a cylinder of radius x is all deposited within a grain diameter of the cylinder. This is a valid assumption in the thin down region where there exist a large number of low energy delta rays, and the track profile is relatively smooth. In the high energy region the maximum delta ray range becomes large and some of the energy flowing out of the cylinder will be deposited at distances much greater than a grain diameter away. Therefore, in this region the theoretical width should be too large. This is observed to be the case, see Plates VIII through XI.

Both theories neglect delta ray scattering but Bizzeti argues that his angular distribution, $f(\theta) + f(\pi - \theta) = \frac{1}{2\pi}$, more closely represents the scattered delta rays than the simpler perpendicular ejection hypothesis of Lonchamp.

One of the conceptually satisfying but experimentally unverified aspects of Bizzeti's theory is the assumption that the track width is the sum of two numbers λ_1 , and λ_0 , where λ_0 is dependent only upon the grain diameter, d , and the growth factor, g , and in particular is independent of λ_1 , the basic track width ($\lambda_0 = (g+1)d$). This assumption is the basis for the normalization procedure described before. However, the normalization to a certain depth did not completely eliminate the discontinuities for some of the heavier tracks. This suggests that width variations due to differences in development are not strictly additive

but probably depend upon the track width. An explanation for this might be that the grains do not grow through each other as implied in Plate III, Figure 1, but in reality displace each other in the growth process (see Powell (18)).

APPLICATION OF THE THEORIES TO POLES

Lonchamp's Theory

To translate Lonchamp's track width theory to poles we return to Equation (6),

$$2r = 4.2 \times 10^{-2} \left[\frac{1020}{\frac{m_0 375 \beta^2}{Z^2} + \frac{1}{\beta^2}} \right]^{1.72} \quad (6)$$

Recall that this equation gives the width of a charged particle track, as a function of β . It can be shown (3) that the equation for delta ray distribution gives the delta ray distribution for monopoles if Ze^2 is replaced $eg\beta$ where g , the pole strength, is $\frac{\hbar c}{2e}$ or $68.5e$. Making this substitution in Equation (6) we obtain,

$$\text{Width} = 4.2 \times 10^{-2} \left[\frac{1020}{\frac{m_0 375 e^2}{g^2} + \frac{1}{\beta^2}} \right]^{1.72} \quad (21)$$

which represents the width of a monopole track. Taking g , the pole strength, to be $68.5e$ this becomes

$$\text{Width} = 4.2 \times 10^{-2} \left[\frac{1020}{\frac{m_0 375}{(68.5)^2} + \frac{1}{\beta^2}} \right]^{1.72} \quad (22)$$

If n_0 , the number of delta rays required for clogging, is taken to be 400 this reduces to

$$\text{Width} = 4.2 \times 10^{-2} \left[\frac{1020}{32 + \frac{1}{\beta^2}} \right]^{1.72} \quad (23)$$

Using range- β values for different pole masses given in Table 1, Appendix I, we can plot width versus range for different pole masses. This has been done in Plate XII.

Bizzeti's Theory

If, as before, we replace Ze^2 by $eg\beta$ in the delta ray distribution formula and proceed in the same manner as for charges, we obtain corresponding to Equation (12),

$$E(k) = \frac{K}{e^2} \frac{g^2}{r} I\left(\frac{r}{R}\right) = \frac{K}{e^2} \frac{g^2}{2} \int_0^R \frac{\bar{w}(k, r)}{w(r)} \frac{dr}{r} \quad (24)$$

and corresponding to Equation (20)

$$\frac{68.5e}{\sqrt{R}} = \phi' \left(\frac{\lambda - \lambda_0}{2R} \right) = \left[\frac{2\pi \times E^*}{\left(\frac{K}{e^2} \right)} \frac{r}{R} \frac{1}{I\left(\frac{r}{R}\right)} \right]^{\frac{1}{2}} \quad (25)$$

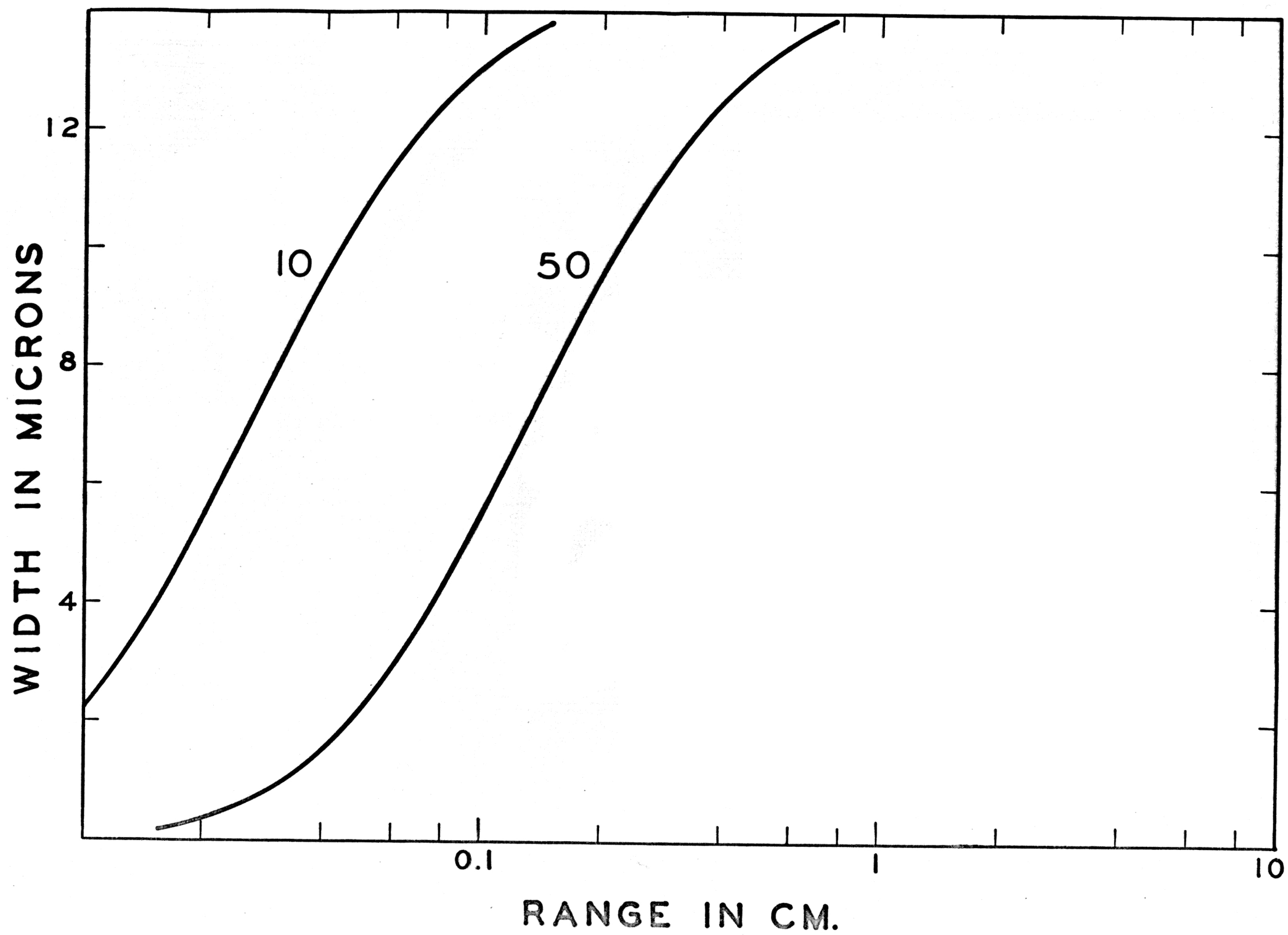
or

$$\frac{68.5}{\sqrt{R}} = \phi \left(\frac{\lambda - \lambda_0}{2R} \right) = \left[\frac{2\pi \times E^*}{K} \frac{r}{R} \frac{1}{I\left(\frac{r}{R}\right)} \right]^{\frac{1}{2}} \quad (26)$$

EXPLANATION OF PLATE XII

Lonchamp's theory applied to monopoles of 10 and 50 amu. Range is directly proportional to mass, so to obtain the width versus range curves for any arbitrary mass M , multiply the range values for the 10 amu curve by $M/10$.

PLATE XII



Now since $\frac{\lambda - \lambda_0}{2R}$ versus $\frac{z}{\beta\sqrt{R}}$ has already been plotted in Plate IV we can relabel the abscissa $68.5/\sqrt{R}$ to yield a plot of $\lambda - \lambda_0/2R$ versus $68.5/\sqrt{R}$.

Following the same procedure as for charges the range- β table for poles (see Table 1, Appendix I) is used to calculate R for different ranges, thus enabling us to obtain $\lambda - \lambda_0$ versus range from the plot of $\frac{\lambda - \lambda_0}{2R}$ versus $\frac{68.5}{\sqrt{R}}$. Plots of $\lambda - \lambda_0$ versus range for poles with different pole masses are given in Plate XIII.

CONCLUSIONS

Careful examination of the theoretical curves, Plates I and V, and the experimental data, Plates VIII, IX, X, XI, indicates that in general where one theory predicts a width that is too small, the other predicts a width too large. We can say then that at all ranges the width can be bracketted using the two theories.

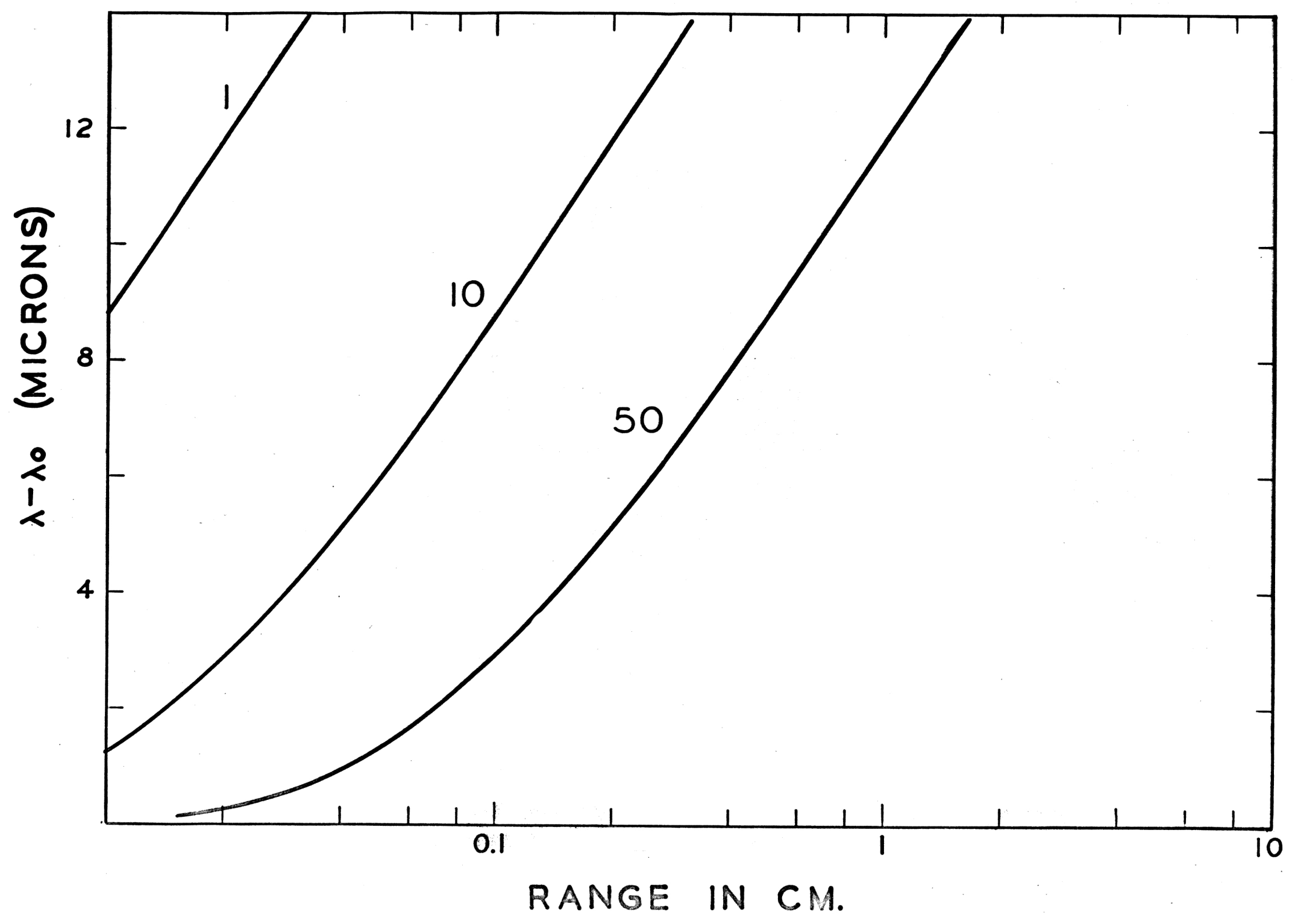
It is reasonable to suppose that the width of a monopole track will also be bracketted by the two theories. Moreover since both theories predict approximately the same width versus range curves, differing mainly in the pole mass, see Plates XII, XIII, we are confident that the theory for poles is at least qualitatively correct.

There are two characteristics of pole tracks that should in most cases make them easily identifiable. First, if a pole

EXPLANATION OF PLATE XIII

Bizzeti's theory applied to monopoles of 1, 10, 50, amu. Range is directly proportional to mass, so to obtain the width versus range curves for any arbitrary mass M , multiply the range values for the 10 amu curve by $M/10$.

PLATE XIII



track has a maximum width, it is greater than 12 microns and occurs at relativistic velocities, out of the domain of our calculations. This reinforces the "wedge shaped" criterion for identification first suggested by Katz and Parnell (14). Second the pole track will be thin (less than a micron wide) up to a certain range depending upon the mass and will then abruptly begin to thicken and take on a wedge shaped profile. These two characteristics will make poles of mass greater than 20 amu or less than 3 amu easily identifiable assuming the last 1,000 microns of track available for measurement. However, very heavy charges will have maximum widths exceeding 12 microns, and will have width variation with range similar to poles of intermediate mass (the exact range of masses depends upon the theory used). For example a pole of mass 6 amu might make a track similar to an ion of charge 37 (using Bizzeti's theory).

At present the theories have been verified only to the extent that qualitative statements can be made. No positive charge identification has been made. Bizzeti's theory appears to be better than Lonchamp's because the curves are closer to the "shape" of the experimental data. Since the theory is assumed to be verified by Bizzeti and Della Corte in the last 200 microns more data should be taken in this region especially in the last 100 microns so that the exact point of deviation of experimental data from the theoretical curve can be estimated.

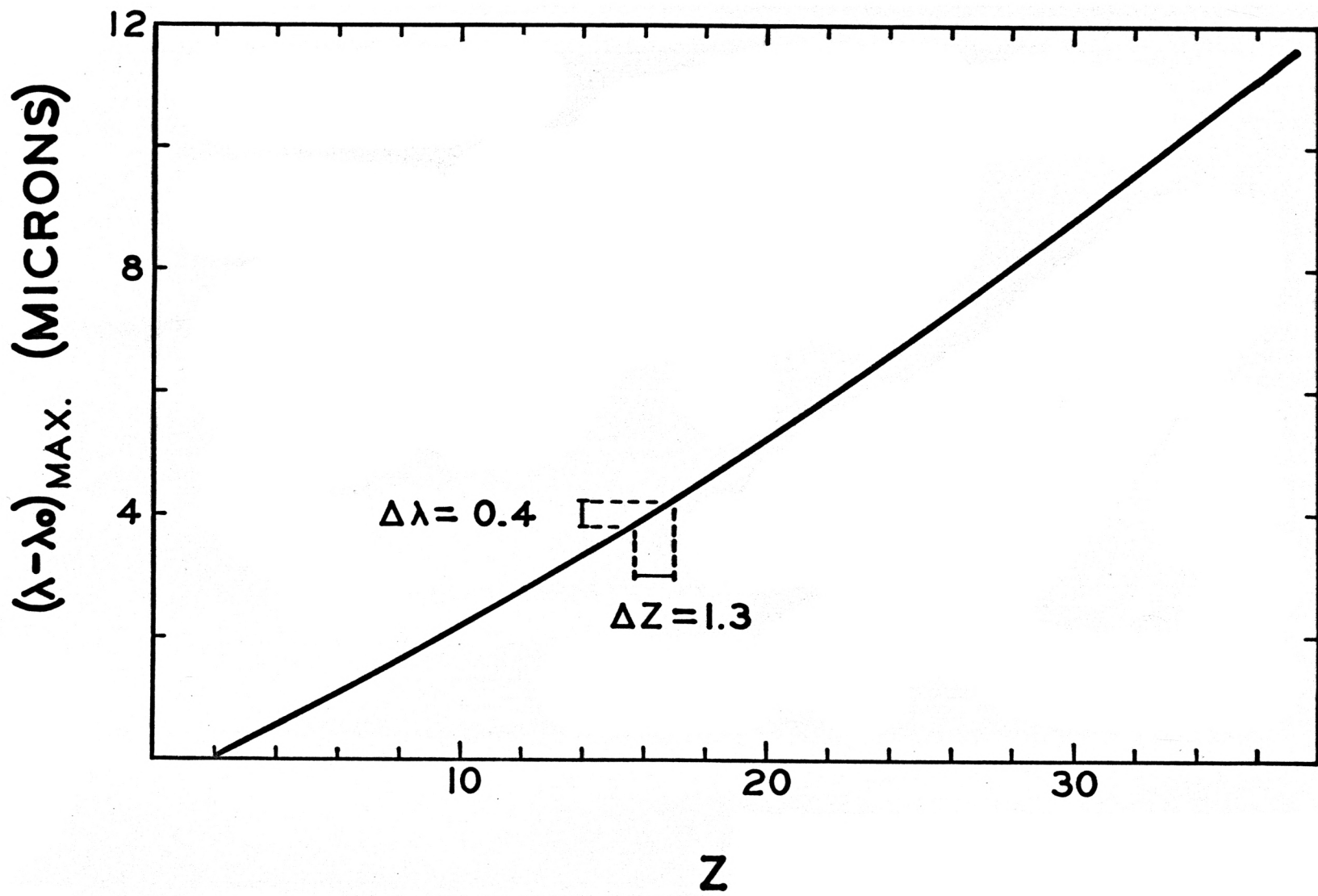
We are now in the process of changing Bizzeti's theory

to make it more accurate in the high energy region. We are changing the basic requirement that there be a constant energy flux per unit area through a cylinder to the requirement that there be a constant amount of energy deposited between two concentric cylinders. This modification in the theory will bring the theoretical width down in the high energy region and should result in much better agreement with experimental data.

If we can succeed in showing the theory of Bizzeti and Della Corte to be correct, then there is an easy and accurate means of heavy charge identification available. In Plate XIV the maximum value of $\lambda - \lambda_0$ has been plotted versus Z . This plot demonstrates that if the maximum width can be determined to within .4 microns then Z can be determined to within 1.3.

EXPLANATION OF PLATE XIV

Plot of maximum value of $\lambda - \lambda_0$ versus Z indicating that an uncertainty of 0.4 in measuring width should theoretically correspond to an uncertainty in Z of 1.3.



ACKNOWLEDGEMENTS

Appreciation is expressed to Dr. Robert Katz for his suggestions, criticisms, and general enthusiasm, and to Earl Hoffman, who did the numerical integrations on the computer and calculated the range- β values for poles (Appendix I).

Appreciation is also expressed to my wife who typed the manuscript.

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APPENDICES

APPENDIX I

Range versus β Calculation for Monopoles

In dealing with charges we used the semi-empirical range energy curves given by Demers (3). To extend the theories to the tracks of poles we will need range-energy or range- β curves for poles. To make this calculation we start with

$$-\frac{dE}{d\psi} = \frac{4\pi Z^2 e^4 N_j}{\beta^2 m_0 c^2} \left[\ln \frac{2\beta^2 c^2 m_0}{\bar{I}_j(1-\beta^2)} - \beta^2 \right],$$

the ionization of a heavy charged particle as given by Bethe (2), where Ze is the charge, and βc is the velocity of the particle, m_0 is the electron mass, N_j is the effective number of electrons per unit volume, and \bar{I}_j is the effective mean ionization potential of the medium. Integrating this equation we can get range as a function of energy,

$$R = \int_0^E \frac{dE}{(-\frac{dE}{d\psi})} = \frac{m_0 c^2}{4\pi e^4 N_j Z^2} \int_0^E \frac{\beta^2 dE}{\left[\ln \frac{2m_0 c^2 \beta^2}{\bar{I}_j(1-\beta^2)} - \beta^2 \right]}. \quad (1)$$

But the relativistic kinetic energy is given by

$$E = Mc^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right)$$

and therefore

$$dE = Mc^2 \beta^2 \gamma^3 d\beta, \quad \text{where} \quad \gamma = (1-\beta^2)^{-\frac{1}{2}}$$

so that

$$R = \int_0^\beta \frac{K' \beta^3 \gamma^3 d\beta}{\left[\ln \frac{2 m_0 c^2 \beta^2 \gamma^2}{\bar{I}_j} - \beta^2 \right]} \quad \text{where} \quad K' = \frac{m_0 c^2 M c^2}{4\pi Z^2 e^4 N_j}$$

This gives the range- β relation for heavy charges.

It can be shown (1) that the ionization, $-\frac{dE}{dv}$, for charges becomes the ionization for monopoles if Ze^2 is replaced by $eg\beta$, where g is the pole strength. The relativistic kinetic energy for poles is, of course, the same as for charges with M replaced by M_p , the pole mass. Making these substitutions we can obtain a range- β relation for poles.

$$R = \int_0^\beta \frac{\frac{M_p}{M} K' \left(\frac{Ze}{g\beta}\right)^2 \beta^3 \gamma^3 d\beta}{\left[\ln \frac{2 m_0 c^2 \beta^2 \gamma^2}{\bar{I}_j} - \beta^2 \right]} = \int_0^\beta \frac{K'' \beta^3 \gamma^3 d\beta}{\left[\ln \frac{2 m_0 c^2 \beta^2 \gamma^2}{\bar{I}_j} - \beta^2 \right]} \quad (2)$$

where

$$K'' = \frac{m_0 c^2 M_p c^2}{4\pi g^2 e^2 N_j}$$

The integrals in Equations (1) and (2) can be done numerically provided we know \bar{I}_j , the effective mean ionization, and N_j , the effective number of electrons per unit volume. We follow a procedure validated for charged particles by Vigneron, (5).

Let the electrons in each unit volume of an absorber be divided into groups according to their binding energies. Considering an emulsion as the absorber, there will be a certain number

of electrons in the bromine K-shell group, a different number in the bromine L-shell group, a different number in the silver M-shell group, and so on for all the shells of all the elements present in the emulsion. As a heavy particle passes through the absorber, the only significant energy transfer is to those electrons which have binding energies less than $2m_0c^2\beta^2\gamma^2$, the maximum energy transferable to an electron by a heavy particle with velocity βc . The groups with binding energies greater than $2m_0c^2\beta^2\gamma^2$ are not "seen" by the particle, and therefore are not "effective" electrons. N_j then, is a function of β and is decreasing in steps as the particle slows down, and different groups of electrons become ineffective. The effective mean ionization potential or effective mean binding energy, \bar{I}_j , will also be a function of β , since the groups that drop from view as the particle slows down will all have greater binding energies than the groups remaining. In our calculations we will consider only the ten most tightly bound groups of electrons in the emulsion.

To calculate $N_j(\beta)$ we take known x-ray absorption edges to be the binding energies and calculate the β at which each of the ten groups considered drops from view. By using Ilford G-5 composition at a density of 3.815 grams/cm³ and N_0 the number of electrons per unit volume as given by Shapiro (4), we can calculate N_j for different β intervals.

To calculate $\bar{I}(\beta)$ let us first consider

$$-\frac{dE}{d\psi} = \frac{4\pi z^2 e^4}{m_0 c^2 \beta^2} N \left(\ln 2m_0 c^2 \beta^2 \gamma^2 - \beta^2 - \ln I \right) \quad (3)$$

the ionization of a heavy particle passing through a substance having electrons of only one binding energy I . For a substance with k different binding energies this becomes

$$-\frac{dE}{d\psi} = \sum_{i=j+1}^k \left(-\frac{dE}{d\psi} \right)_i = \sum_{i=j+1}^k \frac{4\pi z^2 e^4}{m_0 c^2 \beta^2} N_i \left[\ln 2m_0 c^2 \beta^2 \gamma^2 - \beta^2 - \ln I_i \right]$$

where each electron in the i th group, consisting of N_i electrons, has binding energy I_i . The sum runs from $j+1$ to k , because in general the electrons in the first j groups are too tightly bound to be ionized. Rewriting this we have

$$-\frac{dE}{d\psi} = \frac{4\pi z^2 e^4}{m_0 c^2 \beta^2} \left[\ln 2m_0 c^2 \beta^2 \gamma^2 - \beta^2 \right] N_j - \frac{4\pi z^2 e^4}{m_0 c^2 \beta^2} N_j \ln \bar{I}_j$$

where

$$N_j = \sum_{i=j+1}^k N_i$$

and

$$N_j \ln \bar{I}_j = \sum_{i=j+1}^k N_i \ln I_i \quad (4)$$

Equation (4) defines mean ionization potential \bar{I}_j of the effective electrons N_j . Note that $N_j(\beta) \ln \bar{I}_j(\beta) = N_0 \ln \bar{I}_0 - (N_1 \ln I_1 + \dots + N_j \ln I_j)$

where I_1, \dots, I_j , are the binding energies of the j most tightly bound groups and are taken to be the x-ray absorption edges given in the Handbook of Chemistry and Physics, N_0, \dots, N_j can be calculated as described above, and \bar{I}_0 has been determined empirically (4).

We are now able to evaluate the integrals for charges and poles and obtain range- β values for each. The values calculated for charges checked with the accepted values given by Shapiro (4) to within 5% indicating that the procedure was valid. The range- β values for poles of mass 1 amu are given in Table I.

Table I. Range-velocity values for a monopole (1 amu).

β	: pole range : (microns)	:	β	: pole range : (microns)
.05	3.1		.55	128
.10	7.9		.60	156
.15	14.1		.65	190
.20	21.5		.70	233
.25	30.5		.75	287
.30	41.1		.80	344
.35	53.4		.85	467
.40	67.9		.90	641
.45	84.7		.95	1016
.50	105			

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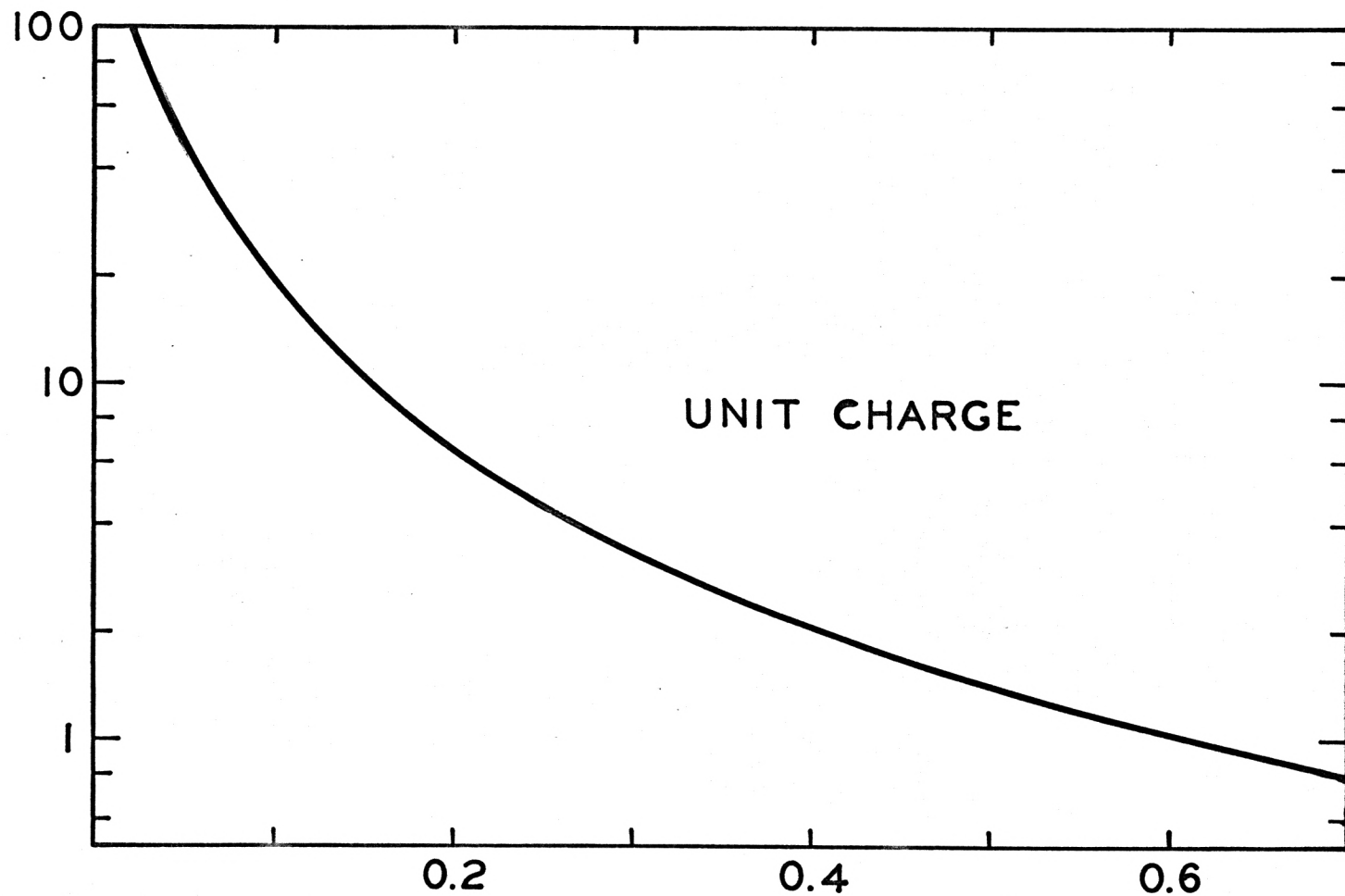
APPENDIX II

The plates that follow have not been referred to explicitly.
They are included here for completeness.

EXPLANATION OF PLATE I

Plot of $-\frac{dE}{d\psi}$ versus β for a unit charge.

$-\frac{dE}{dX}$ (KEV PER MICRON)



$$\beta = \frac{v}{c}$$

PLATE I

EXPLANATION OF PLATE II

Plot of $-\frac{dE}{d\gamma}$ versus β for a unit pole.

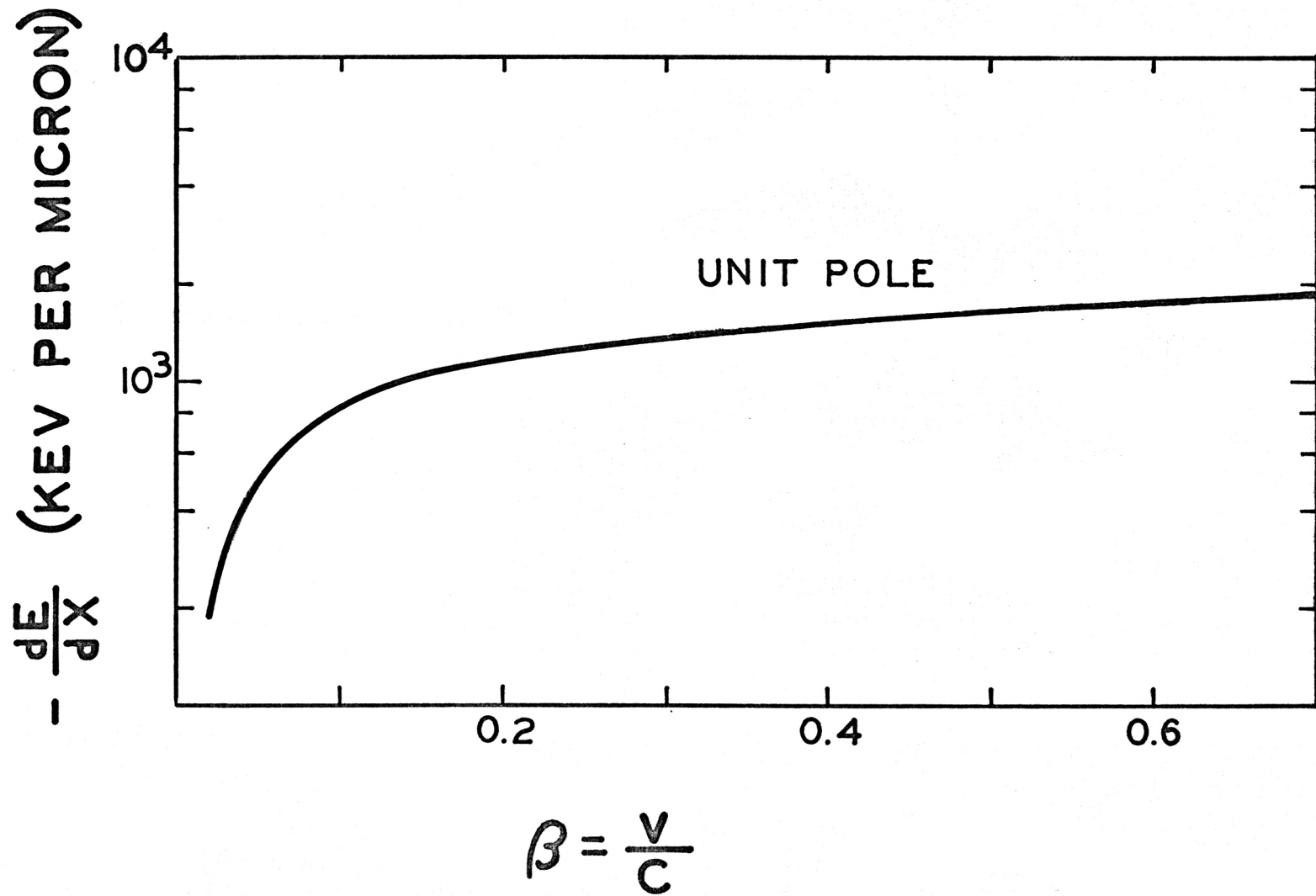


PLATE II

EXPLANATION OF PLATE III

Plot of $-\frac{dE}{dx}$ versus range for poles and
charges of 1 amu.

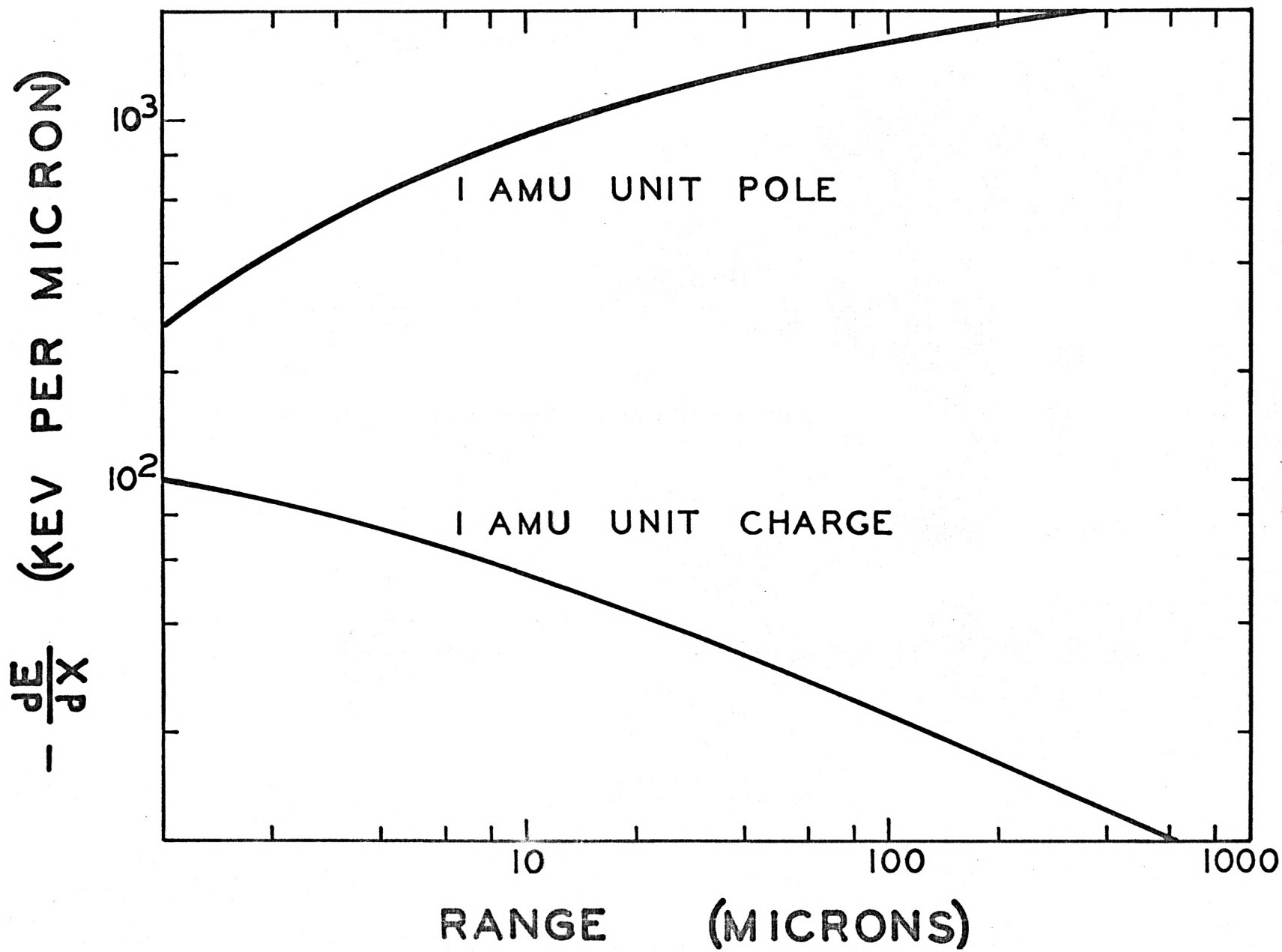
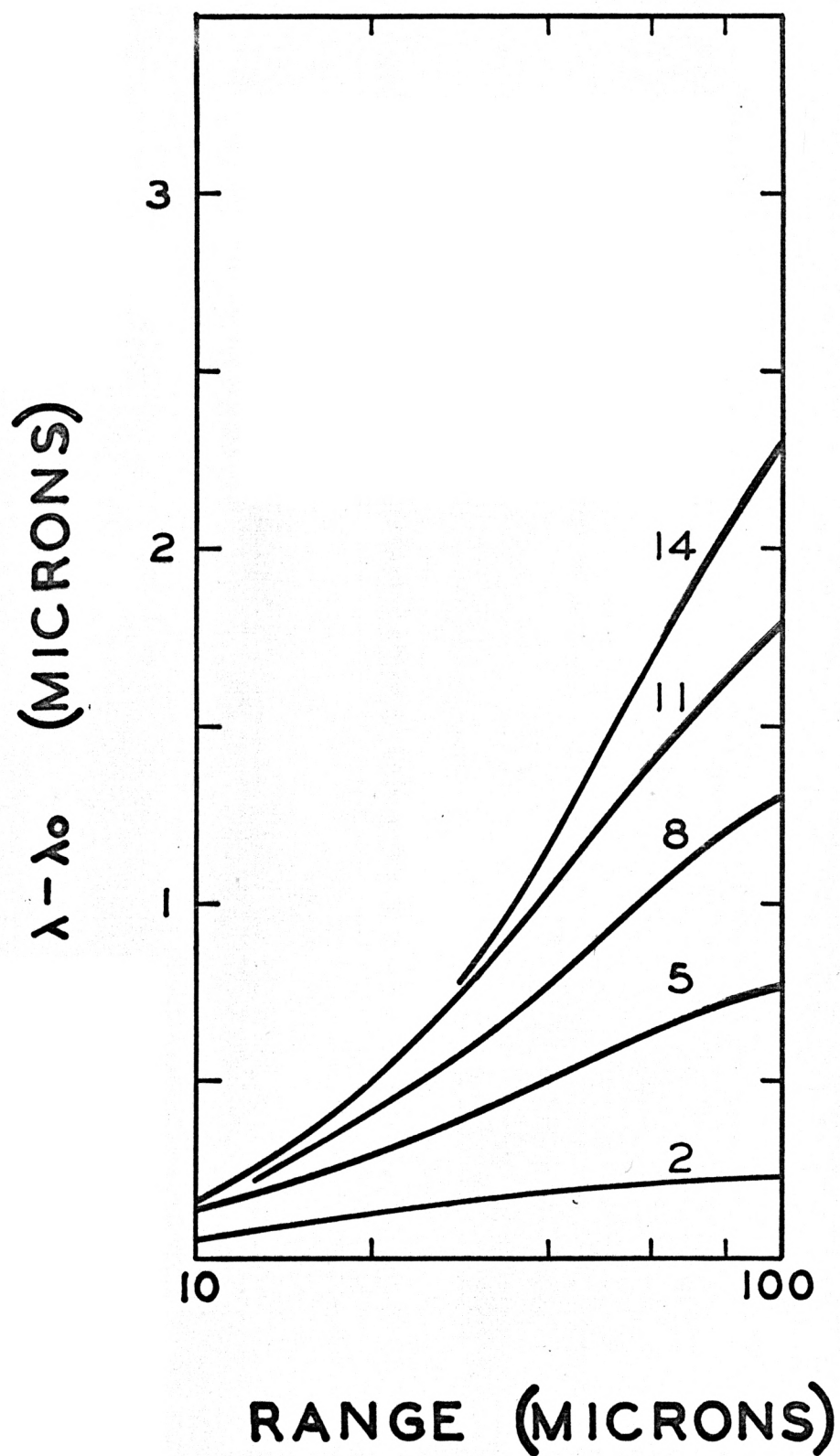


PLATE III

EXPLANATION OF PLATE IV

Plot of $\lambda - \lambda_0$ versus range in 10-100 micron
interval for charges.

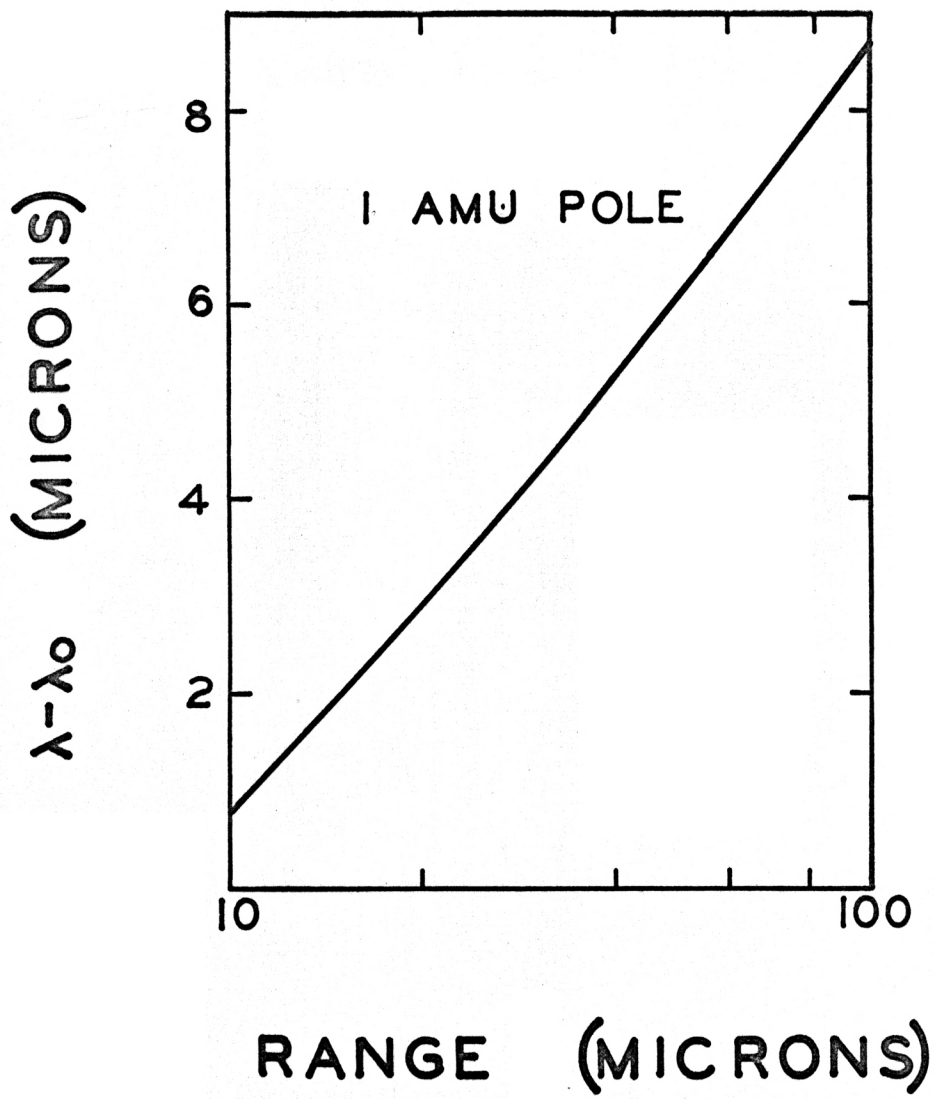
PLATE IV



EXPLANATION OF PLATE V

Plot of $\lambda - \lambda_0$ versus range in 10-100 micron
interval for poles.

PLATE V



EXPLANATION OF PLATE VI

Calculation of width (WD) for various values
of charge Z, with mass (W) and energy (E)
for different values of n_0 (300,400,500).

PLATE VI

```
C  LONCHAMP TRACK WIDTH PROGRAM BUTTS 3/15/1953
C  PUNCH Z,W,E,P,WD
    1  FORMAT(E14.8)
    12  ACCEPT 1,W
        ACCEPT1,Z
    125  ACCEPT1,E
        A=E+W*931.0
        C=931.0*W/A
        B2=1.0-C*C
        DO 4  J=300,500,100
            P=J
            WD=0.042*((1020.0/(P*375.0*B2/(Z*Z)+1.0/B2))**1.72)
    3  FORMAT(I5,I5,F12.1,I6,E18.8)
        K=Z
        L=W
        PUNCH 3,K,L,E,J,WD
    4  CONTINUE
        IF(SENSE SWITCH 3)125,12
        END
```

EXPLANATION OF PLATE VII

Calculation of $\phi = \frac{Z}{\beta \sqrt{R}}$ in Bizzeti and Della
Corte's theory.

PLATE VII

```
C TRACK WIDTH PROGRAM NO. 1 - BUTTS - 2/12/1963
C PRINTS PHI,Z,W,E
  40 FORMAT(E14.8)
  45 ACCEPT40,W
    ACCEPT40,Z
  50 ACCEPT40,E
    A=(E+W*931.0)/(W*931.0)
    R=0.021*(1022.0*(A*A-1.0))**1.72
    C=1.0/A
    B2=1.0-C*C
    PHI=Z/SQRT(B2*R)
150 FORMAT(3H R=E14.8)
    PRINT150,R
130 FORMAT(5H PHI=E14.8,3H Z=F5.1,3H W=E14.8,3H E=E14.8)
    PRINT130,PHI,Z,W,E
    IF(SENSE SWITCH 3)50,45
END
```

EXPLANATION OF PLATE VIII

Integration, using Simpson's rule, of Equation
(19) to obtain $I\left(\frac{x}{R}\right)$ in Bizzeti and Della
Corte's theory.

PLATE VIII

```
C  I(X/R) VS. R/X - HEAVY NUCLEI TRACK WIDTH - HOFFMAN - 2/1/1963
100 FORMAT(E14.8)
110 ACCEPT100,T
    ACCEPT100,H
    A=1.72
    B=1.0/A
    V=1.0/T
    N=((T-1.0)/H)-1.0
    S=0.0
    DO300J=1,N
    W=J
    Y=1.0+W*H
    C=1.0/Y
    Z=C*((1.0-C)**B)*SQRT(1.0-V*V*Y*Y)
    X=W/2.0
    JA=X
    U=JA
    IF(X-U)250,260,250
250 S=S+4.0*Z
    GO TO 280
260 S=S+2.0*Z
280 IF(SENSE SWITCH 1)290,300
285 FORMAT(E14.8,F9.4)
290 PUNCH285,Z,Y
300 CONTINUE
    S=S*(H/3.0)
400 FORMAT(E14.8,F8.3,E14.8)
    PRINT400,S,T,H
    PUNCH400,S,T,H
    GO TO 110
END
```

EXPLANATION OF PLATE IX

Same integration as in Plate VIII using one interval size to integrate $Y=1$ to $Y=10$ and a different interval size elsewhere.

C TWO PART INTEGRATION - HOFFMAN - 2/5/1963
C SENSE SWITCH 1 ON FOR VALUES OF Z VS. Y
C INPUT - TYPE T,H1,H2
C OUTPUT - TYPES S,T,H1,H2

```
100 FORMAT(E14.8)
110 ACCEPT100,T
    ACCEPT100,H1
    ACCEPT100,H2
    A=1.72
    B=1.0/A
    V=1.0/T
    N1=(10.0/H1)-1.0
    S1=0.0
    DO 300J=1,N1
        W=J
        Y=1.0+W*H1
        C=1.0/Y
        Z=C*((1.0-C)**B)*SQRT(1.0-V*V*Y*Y)
        X=W/2.0
        JA=X
        U=JA
        IF(X-U)250,260,250
250  S1=S1+4.0*Z
        GO TO 280
260  S1=S1+2.0*Z
280  IF(SENSE SWITCH 1)290,300
285  FORMAT(E14.8,F13.5)
290  PUNCH285,Z,Y
300  CONTINUE
    CA=1.0/11.0
    ZA=CA*((1.0-CA)**B)*SQRT(1.0-V*V*121.0)
    S1=S1+ZA
    S1=S1*(H1/3.0)
    N2=((T-11.0)/H2)-1.0
    S2=0.0
    DO 400J=1,N2
        W=J
        Y=11.0+W*H2
        C=1.0/Y
        Z=C*((1.0-C)**B)*SQRT(1.0-V*V*Y*Y)
        X=W/2.0
        JA=X
        U=JA
        IF(X-U)350,360,350
350  S2=S2+4.0*Z
        GO TO 380
360  S2=S2+2.0*Z
380  IF(SENSE SWITCH 1)390,400
390  PUNCH285,Z,Y
400  CONTINUE
    S2=S2+ZA
    S2=S2*(H2/3.0)
    S=S1+S2
500  FORMAT(E14.8,F13.5,E14.8,E14.8)
    PRINT500,S,T,H1,H2
    GO TO 110
END
```

EXPLANATION OF PLATE X

This program calculated mean ionization potential $P(J)$, effective electron density $I(J)$, and β for which the mean ionization potential changes $B(J)$. $A(J)$ is the number of electrons in a given energy level (shell). $AI(J)$ is the x-ray absorption edge energy for a given shell. This program works only for one input card since $P(0)$ and $T(0)$ are normalized within the program.

PLATE X

```

C  VARIATION IN THE MEAN IONIZATION POTENTIAL - HOFFMAN - 3/27/1963
      DIMENSION A(11),P(11),T(11),AI(11),B(10)
1    FORMAT(E7.5,E7.5,E7.5,E7.5,E7.5,E7.5,E7.5,E7.5,E7.5,E7.5)
11   READ 1,A(1),A(2),A(3),A(4),A(5),A(6),A(7),A(8),A(9),A(10)
111  READ 1,AI(1),AI(2),AI(3),AI(4),AI(5),AI(6),AI(7),AI(8),AI(9),AI(10)
      DO 2 J=1,10
        A(J)=A(J)*1.0E+22
        AI(J)=AI(J)*1.0E+03
2    CONTINUE
      P(0)=323.0
      T(0)=1.045E+24
      DO 3 J=1,10
        T(J)=T(J-1)-A(J)
        P(J)=EXP((T(J-1)*LOG(P(J-1))-A(J)*LOG(AI(J)))/T(J))
        B(J)=SQRT(AI(J)/1022000.0)
3    CONTINUE
      DO 5 J=1,10
4    FORMAT(F6.1,E16.8,E16.8,F10.1,F8.4)
        PUNCH 4,P(J),T(J),A(J),AI(J),B(J)
5    CONTINUE
      B(0)=1.0
      PUNCH 4, P(0),T(0),B(0),B(0),B(0)
      GO TO 11
      END

```

DATA

.0114	2.03	2.018	0.0456	8.12	.0272	8.072	.1026	18.27	1.874
33.164	25.535	13.48	5.187	3.828	2.469	1.79	1.07	.743	.532

EXPLANATION OF PLATE XI

This is output of program in Plate X. Upper set of data is that given by Vigneron. Lower set is taken from Handbook of Chemistry and Physics.

PLATE XI

DATA

+119.5	+.63929200E+24	+.18740000E+23	+530.0	+.0227
+124.6	+.65803200E+24	+.18270000E+24	+710.0	+.0263
+181.9	+.84073200E+24	+.10260000E+22	+1080.0	+.0325
+182.3	+.84175800E+24	+.80720000E+23	+1770.0	+.0416
+222.4	+.92247800E+24	+.27200000E+21	+2460.0	+.0490
+222.6	+.92275000E+24	+.81200000E+23	+3790.0	+.0608
+280.0	+.10039500E+25	+.45600000E+21	+5180.0	+.0711
+280.3	+.10044060E+25	+.20180000E+23	+13500.0	+.1149
+302.6	+.10245860E+25	+.20300000E+23	+25500.0	+.1579
+329.8	+.10448860E+25	+.11400000E+21	+33200.0	+.1802
+330.0	+.10450000E+25	+.10000000E+01	+1.0	+1.0000

+113.5	+.63929200E+24	+.18740000E+23	+532.0	+.0228
+118.6	+.65803200E+24	+.18270000E+24	+743.0	+.0269
+176.8	+.84073200E+24	+.10260000E+22	+1070.0	+.0323
+177.2	+.84175800E+24	+.80720000E+23	+1790.0	+.0418
+216.9	+.92247800E+24	+.27200000E+21	+2469.0	+.0491
+217.1	+.92275000E+24	+.81200000E+23	+3828.0	+.0612
+273.8	+.10039500E+25	+.45600000E+21	+5187.0	+.0712
+274.1	+.10044060E+25	+.20180000E+23	+13480.0	+.1148
+296.0	+.10245860E+25	+.20300000E+23	+25535.0	+.1580
+322.8	+.10448860E+25	+.11400000E+21	+33164.0	+.1801
+323.0	+.10450000E+25	+.10000000E+01	+1.0	+1.0000

EXPLANATION OF PLATE XII

Integration of Equation (1), (2), in Appendix I.

PLATE XII

```

C RANGE-ENERGY RELATION FOR CHARGES AND POLES - HOFFMAN - 3/9/1963
C SENSE SWITCH 3 ON FOR EVERY VALUE - OFF FOR EVERY TENTH VALUE
  EE=510.9
  E=4.803E-10
  PE=3.14159
  H=1.05E-27
  ER=1.6E-09
  C=2.998E+10
  S1=0.0
  S2=0.0
22  FORMAT(/1HB,5X,3HCEL,8X,3HPEL,8X,2HCI,9X,2HPI,9X,2HRC,9X,2HRP)
220 PUNCH 22
    Q=0.0
4   FORMAT(E6.1,E16.8,E16.8,E10.1,E8.4)
    READ 4, AI,AN,BAD,BBAD,B
    AI=AI*1.0E-03
    CUK=(0.4*PE*E*E*E*E*AN)/(ER*ER*EE)
    PUK=(0.1*PE*AN*H*H*C*C)/(ER*ER*EE)
    PK=1.822E+06*EE/CUK
    PPK=EE*1.822E+06/PUK
401 FORMAT(I3)
    ACCEPT 401,NO
206 DO 3 J=NO,999
    W=J
    W=W*1.0E-03
    IF(B-W)41,41,207
41  READ 4, AI,AN,BAD,BBAD,B
    AI=AI*1.0E-03
    CUK=(0.4*PE*E*E*E*E*AN)/(ER*ER*EE)
    PUK=(0.1*PE*AN*H*H*C*C)/(ER*ER*EE)
    PK=1.822E+06*EE/CUK
    PPK=EE*1.822E+06/PUK
207 G=SQRT(1.0-W*W)
    G3=G*G*G
    D=LOG((2.0*EE*W*W)/(AI*G*G))-W*W
    CEL=(CUK*D)/(W*W)
    PEL=PUK*D
    CI=(PK*W*W*W)/(D*G3)
    PI=(PPK*W)/(D*G3)
226 RC=(S1+CI/2.0)*0.001
    RP=(S2+PI/2.0)*0.001
23  S1=S1+CI
    S2=S2+PI
    IF(SENSE SWITCH 3)92,93
93  Q=Q+1.0
    IF(Q-10.0)3,92,92
21  FORMAT(F6.4,1X,E10.4,1X,E10.4,1X,E10.4,1X,E10.4,1X,E10.4,1X,E10.
92  PUNCH 21,W,CEL,PEL,CI,PI,RC,RP
6   Q=0.0
3   CONTINUE
    STOP
    END

```

TRACK WIDTHS OF HEAVY IONS AND UNIT
MAGNETIC POLES

by

JESSE JAMES BUTTS, JR.

B. S., Kansas State University, 1961

AN ABSTRACT OF A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Physics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

Electron sensitive emulsions that have been exposed to the cosmic rays show very heavy tracks of ionization generally assumed to be the paths of heavy nuclei. However, it is possible that all of these tracks are not caused by heavy nuclei, but that some are in fact tracks of unit magnetic poles. Admitting this possibility, a simple criterion for pole-charge discrimination is needed. In this work, two track theories are first applied to charged particles, and then translated to monopoles.

Measurements were made on eight heavy tracks by projecting their image onto a screen and tracing their outline in pencil. The area of each segment of track was measured with a planimeter and this area divided by the length to obtain the width.

A theory first given by Lonchamp is found to be only qualitative in explaining the width of heavy ion tracks. This theory has been extended by Katz and Parnell to give the width of a pole track as a function of β . Their work is repeated here and carried a step farther to obtain width as a function of range.

A theory developed and applied by Bizzeti and Della Corte, but only in the last 300 microns of range, has been extended here to ranges of 100,000 microns for charges, and then translated to monopoles. Their theory extended in this manner was found to work well up to ranges of 1,500 microns. It was noted that the maximum width according to Bizzeti's theory is a sensitive function of Z , suggesting a simple and accurate means of heavy ion identification.

For monopoles, both theories predict a wedge shaped track that exhibits no maximum width at nonrelativistic velocities.